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# NAVAL POSTGRADUATE SCHOOL

Monterey, California



## THESIS

A STABILITY ANALYSIS  
OF THE PROPOSED  
CIRCULATION CONTROL ROTOR (CCR) PROTOTYPE

by

John Hendrix Cline

March 1977

Thesis Advisor:

L. V. Schmidt

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## (20. ABSTRACT Continued)

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A Stability Analysis  
of the Proposed  
Circulation Control Rotor (CCR) Prototype

by

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Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL

March 1977

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## ABSTRACT

The rotor system of the proposed XH-2/CCR (Circulation Control Rotor) prototype aircraft and the state variable format of the airframe equations of motion are described. Through a study of the eigenvalues and eigenvectors of the basic airframe, the effects of uncoupling and cross-coupling the helicopter equations of motion were analyzed. The control matrix for the CCR was then generated using an applicable pneumatic lead angle for Coanda blowing. Feedback gains, to give the aircraft acceptable flying qualities, were calculated for the Stability Augmentation System (SAS) and finally the aircraft's sensitivity to changes in pneumatic lead angles were studied. The programs used in the above study (capable of handling up to  $10 \times 10$  matrices) are listed for both the IBM-360 and HP-9830 mini-computer.



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## LIST OF SYMBOLS AND ABBREVIATIONS

$g$	Acceleration due to gravity
$L$	Rolling moment about the x-axis due to aerodynamic torques
$M$	Pitching moment about the y-axis due to aerodynamic torques
$N$	Yawing moment about the z-axis due to aerodynamic torques
$p$	Roll rate, angular velocity about the x-axis (positive right wing down) $p = \dot{\phi}$
$q$	Pitch rate, angular velocity about y-axis (positive nose up) $q = \dot{\theta}$
$r$	Yaw rate, angular velocity about z-axis (positive nose right)
$u$	Linear perturbation velocity along x-axis (positive forward)
$U$	Linear steady-state velocity along the x-axis (positive forward)
$v$	Linear perturbation velocity along y-axis (positive out right wing)
$w$	Linear perturbation velocity along z-axis (positive down)
$X$	Aerodynamic force along x-axis (positive forward)
$Y$	Aerodynamic force along y-axis (positive out right wing)
$Z$	Aerodynamic force along z-axis (positive down)
$\phi$	Phase angle of control system
$\psi$	Blade aximuth angle from aft position





## I. INTRODUCTION

In the early 1970's the David W. Taylor Naval Ship Research and Development Center (DTNSRDC) initiated a research program into the feasibility of incorporating a Circulation Controlled Rotor (CCR) system in a Navy helicopter. The concept of the CCR and of improving airfoil lift-to-drag ratios using Coanda flows had been proven earlier by some of the world's leading aerodynamicist's and as early as 1959 Dorand had published in the Journal of the Helicopter Association of Great Britain an article on the application of a jet flap to control a helicopter rotor (Ref. 1). Studies and tests continued throughout the 1960's with more papers published in both the United States and in Europe on the improved performance and possible applications of a CCR (Ref. 2).

The Aviation and Surface Effects Division of DTNSRDC continued the research with further tests involving detailed pressure measurements of two-dimensional elliptical sections in their 15 x 20-inch subsonic wind tunnel. These tests reconfirmed that extremely high lift-to-drag ratios could be achieved by tangentially injecting air through a slot in the trailing edge of an airfoil. The results of these tests were incorporated in both two- and four-bladed model rotor systems for evaluation in the DTNSRDC 8 x 10 foot windtunnel (Ref. 3).



The incorporation of a CCR system in a full-size aircraft could conceivably offer other advantages over the conventional rotor system, in addition to the potentially improved aerodynamic performance traits. The conventional rotating mechanical swashplate system would be replaced by a non-rotating pneumatic plate in a plenum chamber located in the blade hub region. Collective control would be accomplished by changing the plenum chamber pressure, which increases or decreases the Coanda blowing equally at all blades via the individual blade supply or collector tubes. Cyclic control would be provided by tilting the pneumatic swashplate so that there is an azimuthal variation in Coanda blowing in each blade. This variation in blowing is a result of the changes in volume of air allowed to the collector tubes because of changes in the gap between the collector tubes and the non-rotating swashplate. This non-rotating swashplate and variations in Coanda blowing would replace the mechanical cyclic feathering required by conventional rotors and therefore eliminate the vibrations caused by this one-per-revolution cyclic mechanical movement. Another important point to recognize is that the CCR concept, with the Coanda blowing, will dictate a torsionally stiff rotor blade or rigid rotor system. This is a result of the disparity between the two lift generation centroids. The center of pressure due to Coanda blowing is near the blade midchord region, while the blade aerodynamic center remains near the rotor blade quarter chord point.



The proposed helicopter, with a simple hub and rigid rotor blades free of flapping and lag hinges, would result in a relatively clean aerodynamic hub system. The reduction in rotor and hub drag would be beneficial to the helicopter from a performance standpoint. The reduction in moving parts in the hub and blade system would also mean a quieter helicopter with a lower vibration level than that of a conventional rotor system. This latter effect has a favorable potential of improving the "ilities" (maintainability and reliability) for helicopter operations.





## II. BACKGROUND

Early in 1973 the Naval Air Systems Command (NAVAIR) contracted the Lockheed Aircraft Company and Kaman Aerospace Corporation to investigate the feasibility of developing a full-scale flightworthy Circulation Controlled Rotor demonstrator aircraft. In the summer of 1974, some twelve months later, both companies returned reports to DTNSRDC and NAVAIR stating that: "the concept, while innovative, is completely safe in operation" (Ref. 4) and "that there is no fundamental flaw or deficiency in the CCR concept and that construction of a full scale CCR helicopter is feasible and practical" (Ref. 5). Lockheed Aircraft proposed the use of its L286/CCR while Kaman suggested "that the Kaman/Navy H-2 aircraft is an ideal test vehicle for the CCR concept" (Ref. 4).

With the additional goal of being able to incorporate a CCR system on any off-the-shelf helicopter with no major airframe or equipment changes, NAVAIR awarded a contract to Kaman Aerospace to "develop, build and test" a prototype CCR vehicle incorporating the use of the Navy/Kaman H-2 aircraft. This technology demonstration aircraft will tentatively be designated as the Navy XH-2/CCR.

Preliminary studies by Kaman promoted the belief that acceptable flying qualities would be sustained with the installed Stability Augmentation System (SAS) of the Kaman



H-2 with only minor changes in the gains of the feedback amplifiers (Ref. 5). Acceptable flying qualities does not necessarily mean all stable roots of the aircraft motion modes, since a weak oscillatory instability with a time-to-double amplitude of greater than three seconds can be tolerated by a proficient rotor-wing aircraft pilot. The objectives of this research was to confirm that Kaman's beliefs were in fact true and to find a suitable feedback law for the SAS of the XH-2/CCR such that within the aircraft's flight envelope the aircraft flying qualities will be acceptable to the evaluation pilot.



### III. DISCUSSION

The study of the helicopter flight dynamics were conducted using the conventional non-dimensionalized state variable format of the aircraft linearized equations of motion (Ref. 6), modified to allow coupling of the longitudinal with the lateral-directional motions. This modification is a fairly elementary record-keeping operation when using state vector formulations. The basic plant matrix, A, of the aircraft linearized equations of motion in the state variable format is given in Appendix A.

The stability derivatives for the XH-2/CCR airframe were computer generated by the contractor using the MOSTAB-HFA program (Ref. 7) modified for the pertinent characteristics of the SH-2F airframe and the XH-2/CCR main rotor system. The flight conditions analyzed were for 1.0g level flight at sea level standard conditions. The aircraft gross weight was given as 11,000 pounds and a rotor tip speed of 615 feet per second (267 RPM) was used throughout the calculations. Stability derivatives were generated for airspeeds of: Zero (hover), 35, 72, 110, and 130 knots. These derivatives were computed in May of 1976 and then updated in November of the same year. The calculations made in this research effort are based on the updated, November 1976, data.





## A. THE PLANT MATRIX

The plant matrix, A, was developed for the longitudinal and lateral-directional components and then the fully-coupled equations of motion using the contractor generated stability derivatives. The plant matrix, A, was partitioned into:

$$A = \begin{bmatrix} A_{11} & | & A_{12} \\ \hline & | & \\ A_{21} & | & A_{22} \end{bmatrix}$$

where  $[A_{11}]$  represented the coefficients of the longitudinal stability derivatives and  $[A_{22}]$  represented the coefficients of the lateral-directional stability derivatives. The cross-coupling stability derivatives were represented by the coefficients of  $[A_{12}]$  and  $[A_{21}]$ .

The homogeneous form of the state equations took on the conventional form of:

$$\dot{\tilde{x}} = \begin{Bmatrix} \dot{x}_1 \\ \hline \ddot{x}_2 \end{Bmatrix} = A \tilde{x} = A \begin{Bmatrix} x_1 \\ \hline x_2 \end{Bmatrix}$$

where the state vector,  $x$ , included the partitioned longitudinal airframe state vector:

$$x_1^T = [u, w, q, \theta]$$



and the lateral-directional airframe state vector respectively:

$$\mathbf{x}_2^T = [p, r, v, \phi]$$

Normally in airframe dynamics the fact that most "fixed wing" aircraft approach symmetric conditions enables one to eliminate the cross coupling terms  $[A_{12}]$  and  $[A_{21}]$  and then to represent the aircraft by the decoupled equations of motion in the longitudinal and lateral-directional modes. Neglecting these cross-coupling terms allows analysis of the individual fourth-order systems and the size and shape of the applicable longitudinal and lateral-directions modes changes only slightly when the more complicated computations are made for the fully coupled eighth-order systems. Unfortunately, helicopters do not enjoy these conditions of symmetry and the effects of completely cross-coupling the longitudinal and lateral-directional equations are quite significant.

The fact that the helicopter had large cross-coupling effects was shown by the significant change in the eigenvalues for the uncoupled and fully-coupled computations and modal identification could not be completed from the results of the cross-coupled forms alone. The conclusion, therefore, was that the stability problem could not be solved by studying only the uncoupled components of the



equations of motion but must be undertaken using the fully coupled, eighth-order equations of motion.

Using the Basic Matrix Control Theory (BASMAT) computer program (Ref. 8) modified for the HP-9830 mini-computer and the IBM-360 digital computer located at the Naval Postgraduate School's W. R. Church Computer Center; calculations were made at the above stated airspeeds and the eigenvalues and eigenvectors of the plant matrix,  $A$ , were obtained for both the uncoupled and fully-coupled equations of motion. The revised programs for both the IBM-360 and the HP-9830 are included as part of this paper in Appendix B and C respectively.

The existence of a longitudinal unstable root in the uncoupled form was confirmed at all calculated speeds and the identification of the different modes in the fully-coupled eighth-order system was completed by slowly introducing the cross-coupling derivatives into the aircraft equations of motion. The eigenvalues and eigenvectors were traced and modes identified by letting  $A_{21}$  and  $A_{12}$  equal zero in the eighth-order system and then slowly increasing their values until they reached the final values of the basic plant matrix,  $A$ . The values of the uncoupled eigenvalues from the fourth-order solutions and the final eighth-order system results are included in Table I.

A modified root-locus is shown in Figures 1 and 2 for the airspeed conditions of hover and 130 knots respectively. The trajectory of the roots is shown as the amount



TABLE I

Eigenvalues of Basic XH-2/CCR Airframe

Note: Uncoupled airframe listing shows four longitudinal roots first followed by four lateral-directional roots.

Hover	(uncoupled)	(coupled)
$\lambda =$	$0.00159 \pm i 0.10077$	$\lambda = -0.28206 \pm i 0.01601$
	$= -0.23713$	$= 0.01867$
	$= -2.64654$	$= -0.05189$
	$= 0.02949 \pm i 0.27626$	$= -0.00634 \pm i 0.20903$
	$= -0.44778$	$= -5.18781 \pm i 2.53260$
	$= -7.71637$	
35 Knots	(uncoupled)	(coupled)
$\lambda =$	$-0.00971 \pm i 0.16470$	$\lambda = 0.01046 \pm i 0.13193$
	$= -0.30077$	$= -0.24564$
	$= -2.37101$	$= -4.50717 \pm i 2.34599$
	$= -0.24771 \pm i 0.81760$	$= -0.27651 \pm i 0.85642$
	$= -0.05339$	$= -0.06194$
	$= -6.61402$	
72 Knots	(uncoupled)	(coupled)
$\lambda =$	$-0.79763$	$\lambda = -0.05862 \pm i 0.15912$
	$= -0.14599$	$= -0.27382$
	$= -0.11719$	$= -4.40993 \pm i 2.33811$
	$= -2.19846$	$= -0.42956 \pm i 1.53113$
	$= -0.41632 \pm i 1.42319$	$= -0.05781$
	$= -0.05296$	
	$= -6.10178$	
110 Knots	(uncoupled)	(coupled)
$\lambda =$	$-0.38109 \pm i 0.26126$	$\lambda = -0.15433 \pm i 0.25617$
	$= 0.31503$	$= 0.36151$
	$= -2.92894$	$= -4.52643 \pm i 2.93431$
	$= -0.54931 \pm i 1.97156$	$= -0.53360 \pm i 2.10332$
	$= -0.04699$	$= -0.04816$
	$= -5.59397$	
130 Knots	(uncoupled)	(coupled)
$\lambda =$	$-0.30712 \pm i 0.29749$	$\lambda = -0.07151 \pm i 0.20758$
	$= 0.31242$	$= 0.41211$
	$= -3.02559$	$= -4.35708 \pm i 2.81962$
	$= -0.63733 \pm i 2.22591$	$= -0.56744 \pm i 2.34661$
	$= -0.05636$	$= -0.08227$
	$= -5.00383$	





FIGURE 1. CROSS-COUPLING EFFECTS UPON AIRFRAME MODES (HOVER)

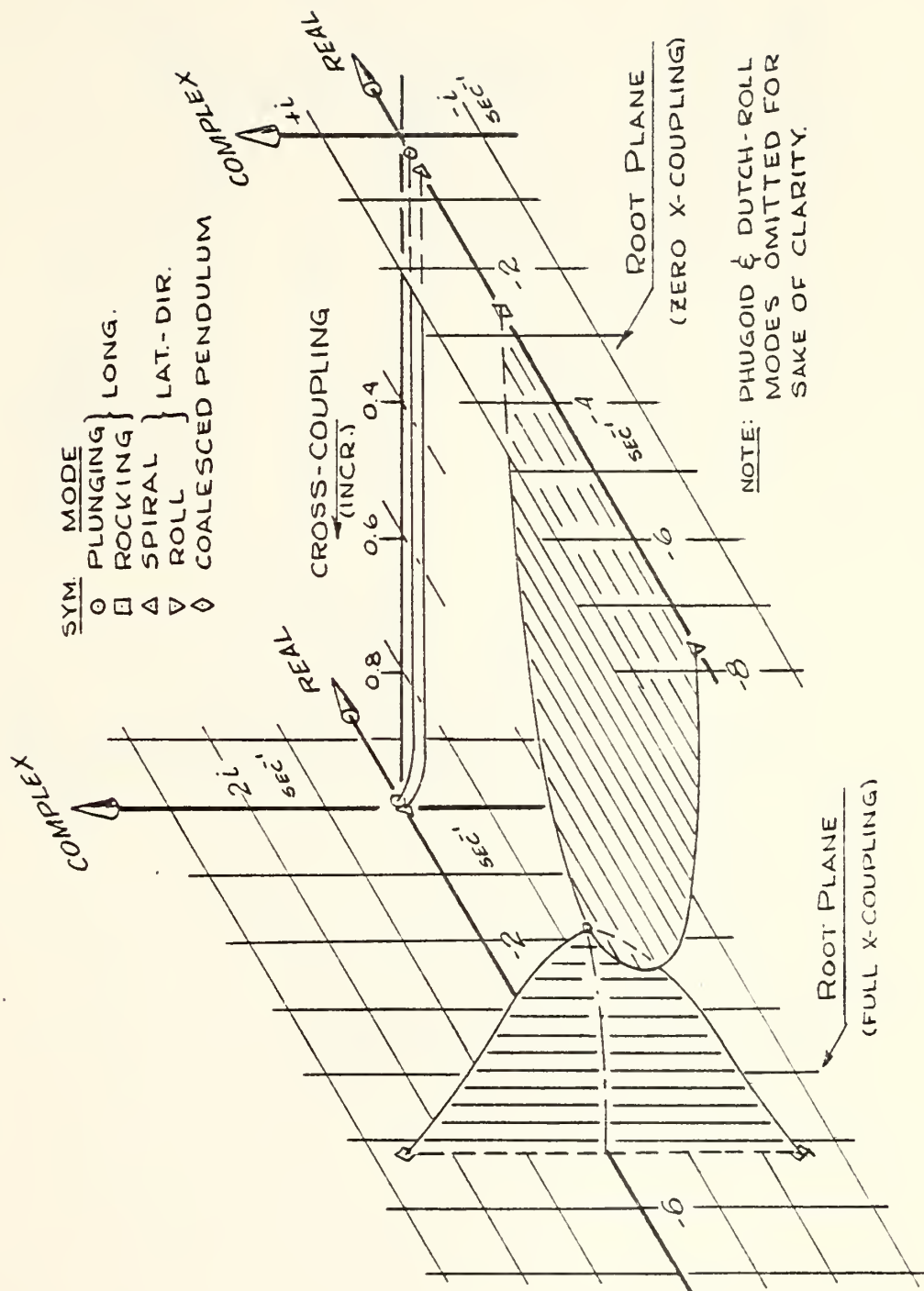
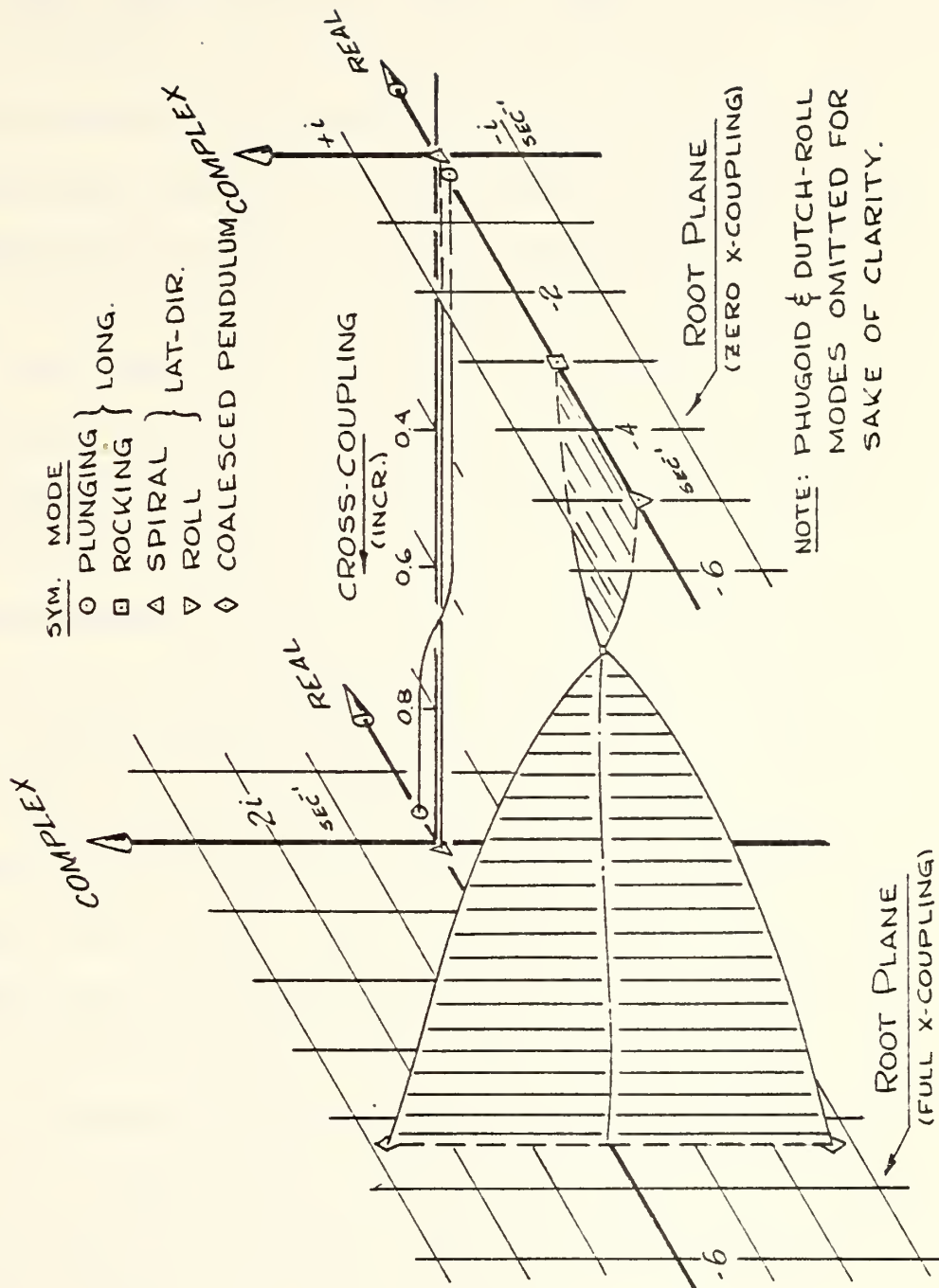




FIGURE 2. CROSS-COUPLING EFFECTS UPON AIRFRAME MODES ( $U = 130$  KNOTS)





of cross-coupling increased from 0.0 to 1.0 with the latter limit corresponding to a fully cross-coupled system. The Dutch-roll and long period oscillatory roots are omitted for sake of clarity, but their values do not vary significantly with cross-coupling as shown in Table I.

The uncoupled non-oscillatory lateral-directional roots may be identified as spiral and roll subsidence roots by both the mode shape and time constants using familiar analogies from fixed wing aircraft. The remaining two non-oscillatory roots from the uncoupled longitudinal degrees of freedom would normally correspond to a short period situation in fixed wing aircraft.

It was observed that the low time constant-real roots, one each from the longitudinal and lateral-directional degrees of freedom respectively remain almost invariant with the amount of cross-coupling until they reach the neighborhood of full cross-coupling. At that time the longitudinal root becomes weakly unstable with a time-to-double amplitude of approximately 37 and 1.68 seconds at hover and 130 knots respectively. The latter situation definitely required improvement by means of stability augmentation. These two real roots could have been expected to coalesce into an oscillatory pair as cross-coupling varied, but possibly the close proximity to the almost invariant oscillatory long period and Dutch-roll roots prevented this action from occurring.



The second real-longitudinal root and the roll subsidence root may be observed to coalesce into a pair of complex conjugate (oscillatory roots) almost mid-range in the cross-coupling. The coalesced roots have a mode shape similar to a pendulum type of motion, but it was noted that this mode (in the fully-coupled situations) was quite heavily damped.

The aircraft mode shapes are defined in Table II for hover and 130 knots velocities in both the uncoupled and fully coupled situations. Without these mode shapes, there would be difficulties involved in interpreting the Characteristic root migrations as shown in Figures 1 and 2. The symbol (⊙) is used on the figures to identify a longitudinal plunging subsidence mode which will be spotted in Table II with the (w) velocity perturbation being the dominant term. This root, which in conventional airframe systems would be combined with the rocking mode root (⊠) to yield the conventional oscillatory short period mode, is the "culprit" which becomes unstable in the fully coupled situation. As will be noted in Figures 1 and 2, the longitudinal rocking and the lateral roll subsidence modes coalesce into an oscillatory pendulum type mode.

## B. THE CONTROL MATRIX

In addition to the definition of the airframe plant, the contractor provided the MOSTAB generated control matrices. The derivative information, presumably the result





TABLE II  
MODE SHAPE SUMMARY

A. Hover Conditions

1. Zero Cross-Coupling

a. Longitudinal-oscillatory long period

$$\lambda = 0.0016 \pm i 0.1008 \quad \text{Period} = 62.3 \text{ sec} \quad T_2 = 435.94 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 1.0000; \text{arg. } 0.0 \text{ deg.} \\ 0.0196; \text{arg. } -116.8 \text{ deg.} \\ 0.0003; \text{arg. } 83.8 \text{ deg.} \\ 0.0031; \text{arg. } -5.4 \text{ deg.} \end{Bmatrix}$$

b. Longitudinal-plunging subsidence

$$\lambda = -0.2371 \quad T_{\frac{1}{2}} = 2.92 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0.0700 \\ 1.0000 \\ 0.0000 \\ 0.0000 \end{Bmatrix}$$

c. Longitudinal-fore/aft rocking subsidence

$$\lambda = -2.647 \quad T_{\frac{1}{2}} = 0.26 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0090 \\ -0.1830 \\ 0.0690 \end{Bmatrix}$$

d. Lateral-Directional-Dutch Roll

$$\lambda = 0.0295 \pm i 0.2762 \quad \text{Period} = 22.8 \text{ sec.} \quad T_2 = 23.4 \text{ sec.}$$

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0025; \text{arg. } 166.4 \text{ deg.} \\ 0.0250; \text{arg. } -34.4 \text{ deg.} \\ 1.0000; \text{arg. } 0.0 \text{ deg.} \\ 0.0090; \text{arg. } 82.4 \text{ deg.} \end{Bmatrix}$$

e. Directional-Spiral subsidence

$$\lambda = -0.448 \quad T_{\frac{1}{2}} = 1.55 \text{ sec.}$$

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0044 \\ -0.1650 \\ 1.000 \\ 0.0099 \end{Bmatrix}$$

f. Lateral-Roll Subsidence

$$\lambda = -7.716 \quad T_{\frac{1}{2}} = 0.090 \text{ sec.}$$

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0294 \\ 0.8669 \\ -0.1296 \end{Bmatrix}$$



TABLE II (Continued)

## A. 2. Full Cross-coupling

## a. Longitudinal-oscillatory long period

$$\lambda = -0.2821 \pm i 0.0160 \quad \text{Period} = 392.7 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000; \text{arg. } 0.0 \text{ deg.} \\ 0.7102; \text{arg. } 14.1 \text{ deg.} \\ 0.0025; \text{arg. } 173.4 \text{ deg.} \\ 0.0089; \text{arg. } -3.3 \text{ deg.} \\ 0.0008; \text{arg. } 176.0 \text{ deg.} \\ 0.0490; \text{arg. } 174.5 \text{ deg.} \\ 0.1924; \text{arg. } 182.6 \text{ deg.} \\ 0.0028; \text{arg. } -0.7 \text{ deg.} \end{Bmatrix} \quad T_1 = 2.46 \text{ sec}$$

$$\frac{T_1}{2}$$

## b. Cross-coupled Pendulum Mode

$$\lambda = -5.1878 \pm i 2.5326 \quad \text{Period} = 2.48 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.6181; \text{arg. } -141.88 \text{ deg.} \\ 0.0798; \text{arg. } 155.40 \text{ deg.} \\ 0.4219; \text{arg. } 2.14 \text{ deg.} \\ 0.0731; \text{arg. } -155.84 \text{ deg.} \\ 0.7618; \text{arg. } -42.92 \text{ deg.} \\ 0.0386; \text{arg. } 177.95 \text{ deg.} \\ 1.0000; \text{arg. } 0.0 \text{ deg.} \\ 0.1320; \text{arg. } 163.10 \text{ deg.} \end{Bmatrix} \quad T_1 = 0.13 \text{ sec}$$

$$\frac{T_1}{2}$$

## c. Lateral-Direction - Coupled Dutch Roll Mode

$$\lambda = -0.0063 \pm i 0.2090 \quad \text{Period} = 30.1 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000; \text{arg. } 0.00 \text{ deg.} \\ 0.0162; \text{arg. } -171.87 \text{ deg.} \\ 0.0014; \text{arg. } 3.04 \text{ deg.} \\ 0.0065; \text{arg. } -88.70 \text{ deg.} \\ 0.0006; \text{arg. } 145.88 \text{ deg.} \\ 0.0126; \text{arg. } 1.69 \text{ deg.} \\ 0.4099; \text{arg. } 33.20 \text{ deg.} \\ 0.0028; \text{arg. } 122.38 \text{ deg.} \end{Bmatrix} \quad T_1 = 109.26 \text{ sec.}$$

$$\frac{T_1}{2}$$

## d. Lateral-

$$\lambda = -0.0519 \quad T_1 = 13.36 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0358 \\ -0.0000 \\ 0.0016 \\ -0.0000 \\ -0.0077 \\ -0.1992 \\ 0.0003 \end{Bmatrix} \quad \frac{T_1}{2}$$



TABLE II (Continued)

## e. Longitudinal

$$\lambda = 0.0187$$

$$T_2 = 37.13 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0224 \\ -0.0000 \\ -0.0006 \\ -0.0000 \\ -0.2016 \\ -0.0001 \end{Bmatrix}$$

## B. 130 Knots

## 1. Zero Cross-coupling

## a. Longitudinal-oscillatory Long Period Mode

$$\lambda = -0.3071 \pm i 0.2975 \quad \text{Period} = 21.1 \text{ sec.} \quad T_{\frac{1}{2}} = 2.26 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0.5812; \text{ arg. } -67.70 \text{ deg.} \\ 1.0000; \text{ arg. } 0.0 \text{ deg.} \\ 0.0026; \text{ arg. } 39.45 \text{ deg.} \\ 0.0061; \text{ arg. } -96.46 \text{ deg.} \end{Bmatrix}$$

## b. Longitudinal -

$$\lambda = 0.3124$$

$$T_2 = 2.22 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.7721 \\ -0.0043 \\ -0.0138 \end{Bmatrix}$$

## c. Longitudinal -

$$\lambda = -3.0256$$

$$T_{\frac{1}{2}} = 0.23 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0.0185 \\ 1.0000 \\ -0.0108 \\ 0.0036 \end{Bmatrix}$$

## d. Lateral-Directional Dutch Roll Mode

$$\lambda = -0.6373 \pm i 2.2259 \quad \text{Period} = 2.8 \text{ sec.} \quad T_{\frac{1}{2}} = 1.09 \text{ sec.}$$

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0185; \text{ arg. } 0.02 \text{ deg.} \\ 1.0000; \text{ arg. } 0.0 \text{ deg.} \\ 0.0107; \text{ arg. } 179.96 \text{ deg.} \\ 0.0036; \text{ arg. } - 0.0 \text{ deg.} \end{Bmatrix}$$



TABLE II (Continued)

## e. Lateral-

$$\lambda = -0.0564$$

$$T_{\frac{1}{2}} = 12.29 \text{ sec.}$$

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} -0.0077 \\ 0.0203 \\ 1.0000 \\ 0.1363 \end{Bmatrix}$$

## f. Lateral-

$$\lambda = -5.0038$$

$$T_{\frac{1}{2}} = 0.14 \text{ sec.}$$

$$\begin{Bmatrix} p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ -0.0381 \\ -0.0674 \\ -0.1999 \end{Bmatrix}$$

## B. 2. Full Cross-coupling

## a. Longitudinal-oscillatory Long Period Mode

$$\lambda = -0.0715 \pm i 0.2076 \quad \text{Period} = 30.3 \text{ sec.} \quad T_{\frac{1}{2}} = 9.69 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000; \text{arg. } 0.0 \text{ deg.} \\ 0.4987; \text{arg. } 10.05 \text{ deg.} \\ 0.0014; \text{arg. } 34.18 \text{ deg.} \\ 0.0064; \text{arg. } -74.83 \text{ deg.} \\ 0.0045; \text{arg. } 167.65 \text{ deg.} \\ 0.0027; \text{arg. } 81.22 \text{ deg.} \\ 0.3260; \text{arg. } 25.33 \text{ deg.} \\ 0.2051; \text{arg. } 883.34 \text{ deg.} \end{Bmatrix}$$

## b. Lateral-Directional - Coupled Dutch Roll Mode

$$\lambda = -0.5674 \pm i 2.3466 \quad \text{Period} = 2.7 \text{ sec.} \quad T_{\frac{1}{2}} = 1.22 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0111; \text{arg. } -109.36 \text{ deg.} \\ 0.2045; \text{arg. } -20.66 \text{ deg.} \\ 0.0023; \text{arg. } -74.39 \text{ deg.} \\ 0.0009; \text{arg. } 88.99 \text{ deg.} \\ 0.0045; \text{arg. } 159.81 \text{ deg.} \\ 0.0110; \text{arg. } -76.86 \text{ deg.} \\ 1.0000; \text{arg. } 0.0 \text{ deg.} \\ 0.0019; \text{arg. } 56.22 \text{ deg.} \end{Bmatrix}$$





TABLE II (Continued)

## c. Cross-couple Pendulum Mode

$$\lambda = -4.3571 \pm i 2.8196 \quad \text{Period} = 2.2 \text{ sec.} \quad T_{\frac{1}{2}} = 0.16 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0.0177; \text{arg. } 12.86 \text{ deg.} \\ 1.0000; \text{arg. } 0.0 \text{ deg.} \\ 0.0214; \text{arg. } -66.01 \text{ deg.} \\ 0.0041; \text{arg. } -3.95 \text{ deg.} \\ 0.0412; \text{arg. } 82.02 \text{ deg.} \\ 0.0066; \text{arg. } -8.08 \text{ deg.} \\ 0.2695; \text{arg. } 37.04 \text{ deg.} \\ 0.0079; \text{arg. } -65.07 \text{ deg.} \end{Bmatrix}$$

## d. Longitudinal-

$$\lambda = 0.4121 \quad T_2 = 1.68 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} -0.7732 \\ 1.0000 \\ 0.0006 \\ 0.0014 \\ -0.0031 \\ -0.0014 \\ -0.0342 \\ -0.0075 \end{Bmatrix}$$

## d. Lateral

$$\lambda = -0.0823 \quad T_{\frac{1}{2}} = 8.42 \text{ sec.}$$

$$\begin{Bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{Bmatrix} = \begin{Bmatrix} 1.0000 \\ 0.0781 \\ -0.0000 \\ 0.0009 \\ -0.0041 \\ 0.0076 \\ 0.5428 \\ 0.0504 \end{Bmatrix}^{\frac{1}{2}}$$



of pneumo-dynamic modeling in the hub and blade blowing sections, was supplied as airframe forces and moments per unit (3060 psf) pressure variation at the pneumatic swashplate referenced to the aircraft axis. The cyclical variation of the plenum pressure upon control was given by:

C (2) .. The coefficient of the cosine  $\psi$  type variation in the control matrix.

C (3) .. The coefficient of the sine  $\psi$  type variation in the control matrix.

where,  $\psi$ , represents the blade azimuthal angle. One can visualize the longitudinal and lateral cyclic controls as rotating the pneumatic swashplate about an orthogonal set of axes that leads the blade azimuthal angle by some angle,  $\phi$ , in an analogous manner to the practise on mechanical swashplates. Then, as shown in Figure 3, the apparent longitudinal and lateral cyclic control matrices become, by a coordinate rotation, as follows:

$$B (1) = C (2) \cos \phi - C (3) \sin \phi$$

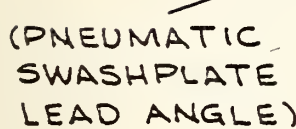
$$B (2) = C (2) \sin \phi + C (3) \cos \phi$$

where B(1) and B(2) are the longitudinal and lateral cyclic control matrices respectively. No attempt was made to relate these control matrices to actual control stick motions, although reasonable estimates could have been made.

The criteria employed in selecting the control lead angle was the following:



↓ U


$$\begin{Bmatrix} B(1) \\ B(2) \end{Bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{Bmatrix} C(2) \\ C(3) \end{Bmatrix}$$

$$\begin{Bmatrix} B(1) \\ B(2) \end{Bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{Bmatrix} C(2) \\ C(3) \end{Bmatrix}$$



- o Longitudinal cyclic .. produced negligible rolling moment.
- o Lateral cyclic .. produced negligible pitching moment.

Although absolute satisfaction of these constraints concurrently with one choice of lead angle,  $\phi$ , was not physically realizable, it was remarkable that a single value of lead angle,  $\phi = 40$  degrees, provided a satisfactory solution in an engineering sense.

Table III lists the variation of pneumatic lead angle versus airspeed for satisfaction of the longitudinal and lateral cyclic constraints respectively. The selection of forty (40) degrees as an engineering answer is in accord with independent analysis done by Kaman Aerospace. The tabulation of the B(1) and B(2) control matrices for a pneumatic lead angle of forty (40) degrees are presented in Table IV. Inspection of the fifth row of B(1) and third row of B(2) provides an indication of the reasonableness of the solution.

The effects of plus and minus five (5) degree changes in pneumatic lead angle will be described during the analysis. This type of sensitivity analysis will provide an indication of the airframe stability root sensitivity in the compensated mode.

### C. THE FEEDBACK LAW

The study of the impact of various feedback control laws for reducing or removing the longitudinal instability





TABLE III

Calculations of the Pneumatic Lead Angle vs. Airspeed

$$B(1) = C(2) \cos \phi - C(3) \sin \phi$$

$$\phi_{B1} = \arctan \frac{C(2)_L}{C(3)_L}$$

$$B(2) = C(2) \sin \phi - C(3) \cos \phi$$

$$\phi_{B2} = \arctan \frac{-C(3)_N}{C(2)_N}$$

HOVER	$\phi_{B1} = 42.32^\circ$
-------	---------------------------

	$\phi_{B2} = 43^\circ 50'$
--	----------------------------

35 KNOTS	$\phi_{B1} = 37.16^\circ$
----------	---------------------------

	$\phi_{B2} = 39.36^\circ$
--	---------------------------

72 KNOTS	$\phi_{B1} = 40.87^\circ$
----------	---------------------------

	$\phi_{B2} = 33.09^\circ$
--	---------------------------

110 KNOTS	$\phi_{B1} = 39.83^\circ$
-----------	---------------------------

	$\phi_{B2} = 36.63^\circ$
--	---------------------------

130 KNOTS	$\phi_{B1} = 40.61^\circ$
-----------	---------------------------

	$\phi_{B2} = 42.56^\circ$
--	---------------------------



TABLE IV

Coefficients of the Control Matrix  $B_{1x}$   
and  $B_{2x}$  for pneumodynamic lead angle  
of (40) degrees

	HOVER	35 KTS	72 KTS	110 KTS	130 KTS
$B_{11}$	-12.96	-11.13	-8.474	-7.593	-5.121
$B_{21}$	-8.697	-13.08	-9.193	-22.13	-27.61
$B_{31}$	9.356	7.565	6.911	7.223	6.726
$B_{41}$	0.0	0.0	0.0	0.0	0.0
$B_{51}$	1.164	-1.124	0.295	-0.053	0.174
$B_{61}$	-0.267	-0.055	-1.446	-0.905	-1.098
$B_{71}$	4.897	2.816	2.842	3.049	2.030
$B_{81}$	0.0	0.0	0.0	0.0	0.0

$B_{12}$	5.486	3.860	2.086	0.639	1.711
$B_{22}$	-0.765	5.042	2.296	11.21	11.53
$B_{32}$	-0.572	0.084	0.837	0.425	0.304
$B_{42}$	0.0	0.0	0.0	0.0	0.0
$B_{52}$	28.72	22.68	19.52	18.30	16.52
$B_{62}$	-0.106	-0.643	0.521	0.965	0.585
$B_{72}$	12.94	10.36	9.244	10.60	9.996
$B_{82}$	0.0	0.0	0.0	0.0	0.0



began with an attempt to vary the available longitudinal feedback gains and to investigate the eigenvalues and eigenvectors as these gains were varied.

The modified plant matrix,  $A'$ , was developed in the traditional matrix manner in the uncoupled and fully-coupled state variable format where:

$$A' = A - Bk$$

When only longitudinal cyclic control is considered, the control effectiveness matrix  $[B]$  becomes an eight-by-one matrix while the feedback gain coefficient matrix  $[k]$  becomes a one-by-eight matrix. The matrix product,  $Bk$ , is an eight-by-eight matrix.

$$B^T = [U, W, Q, \theta, P, R, V, \phi]$$

$$k = [k_u, k_w, k_q, k_\theta, k_p, k_r, k_v, k_\phi]$$

The earlier confirmation that the unstable root was primarily associated with the longitudinal airframe modes was the reason for only employing feedback in the longitudinal cyclic control when developing the modified plant matrix,  $[A']$ . An arbitrary set of moderately damped oscillatory stable roots were selected:

$$\lambda_{1,2} = 1.68 \pm i 8.21$$



which corresponded to:

$$\omega_n = \text{undamped natural frequency} = 8.38 \text{ sec}^{-1}$$

$$\zeta = \text{dimensionless damping ratio} = 0.2$$

Since it had been established in the basic plant matrix that the short period mode was the dominant instability, the arbitrarily selected second-order system (a form of modal control) was applied to the augmented two-by-two matrix,  $A'$ , and the closed form solution calculated to yield values of  $k_w$  and  $k_q$ . Gain values of  $k_w = 0.05$  and  $k_q = 0.22$  were determined to yield the desired results but the application of these gains alone, to the uncoupled four-by-four matrix and the fully-coupled eight-by-eight state variable problem failed to produce favorable results and the instability remained with the aircraft matrix at all of the calculated speeds.

The search for the acceptable feedback law using gains of  $k_w$  and  $k_q$  continued using the HR-9830 with a further modified BASMAT program that would automatically search for acceptable feedback gains in the range of  $k_w$  and  $k_q$  equal to minus one (-1) to plus one (+1.0). Although values of  $k_w$  and  $k_q$  could be found that would drive the augmented matrix stable (negative real parts of the eigenvalues) at each calculated speed, these values were not sequentially related to speed and, furthermore, were random in nature, often changing signs more than once as speed increased from hover.





The decision was then made to employ pitch attitude and pitch rate ( $k_\theta$  and  $k_q$ ) feedback respectively based on the knowledge that both pitch and pitch rate information were presently available in the H-2 aircraft. Programs existed for both the IBM-360 and the HP-9830 for accomplishing this search for acceptable feedback gains of  $k_q$  and  $k_\theta$ , but while the IBM-360 was much faster in the actual computations (approximately twenty seconds of CPU time were required for the computations of the eigenvalues and eigenvectors for one set of plant, control and feedback gain coefficients vice twenty-five minutes for the HP-9830), the HP-9830 allowed for a much more convenient search. The HP-9830 allowed the programmer to make "in-line" decisions on changes in the feedback gains based on the previous results with approximately thirty minutes between output results. This removed the problem of the long delays encountered because of the turnaround time of the IBM-360. Typical turnaround times were two to five hours depending upon the computer usage at the time of program input. This long turnaround time was a result of the low job priority assigned the program by the computer center, a result of the complexity of the program and the large amount of core memory required.

The rationale of feeding back  $k_q$  and  $k_\theta$  proved fruitful for values of:

$$k_q = 0.45 \quad \text{and} \quad k_\theta = 0.85$$



or the feedback matrix taking the form of:

$$k = [0, 0, 0.45, 0.85, 0, 0, 0, 0]$$

producing stable eigenvalues at all speeds calculated with the exception of hover. These gains did leave an unstable oscillatory root in hover, with a time to double amplitude of 18.15 seconds, over six times the three second minimum time to double amplitude allowed.

#### D. PNEUMATIC LEAD ANGLE SENSITIVITY CHECK

Concern had been shown over the possibility that variations in the pneumatic lead angle ( $\phi$ ) blowing could cause dramatic changes in the stability characteristics of the aircraft. It was recognized that changes in pneumatic lead angle would result in direct changes to the control matrix, B. In order to study the sensitivity to changes in lead angle, new values of the control matrix were computed for pneumatic lead angles of thirty-five (35) and forty-five (45) degrees. The results of these calculations are listed in Table VI. The IBM-360 program was further modified to compute the new eigenvalues and eigenvectors using the basic plant matrix, A, the computed feedback gains, k, given above, and allowing the longitudinal pneumatic control matrix, B, to vary for values of  $\phi$  equal to thirty-five and forty-five degrees. The computed results



TABLE V

Eigenvalues of the augmented matrix,  $A'$ , at  
velocities of: Hover, 35, 72, 110 and 130 Knots

Hover:

$$\begin{aligned}\lambda &= -0.00019 \\ &= -0.24473 \\ &= -0.47775 \\ &= -0.97459 \\ &= 0.03819 \pm i 0.27676 \\ &= -6.78748 \pm i 3.60274\end{aligned}$$

35 Knots:

$$\begin{aligned}&= -0.01709 \\ &= -0.04164 \\ &= -0.24762 \pm i 0.81412 \\ &= -0.67857 \pm i 0.13139 \\ &= -5.67357 \pm i 3.42617\end{aligned}$$

72 Knots:

$$\begin{aligned}&= -0.01445 \\ &= -0.07094 \\ &= -0.63074 \pm i 0.37790 \\ &= -0.43457 \pm i 1.47940 \\ &= -5.45309 \pm i 3.07803\end{aligned}$$

110 Knots:

$$\begin{aligned}&= -0.00326 \\ &= -0.04764 \\ &= -0.49037 \pm i 0.55734 \\ &= -0.54836 \pm i 2.06899 \\ &= -5.61868 \pm i 3.46986\end{aligned}$$

130 Knots:

$$\begin{aligned}&= -0.01401 \\ &= -0.07035 \\ &= -0.39270 \pm i 0.61107 \\ &= -0.59046 \pm i 2.34511 \\ &= -5.31913 \pm i 3.22033\end{aligned}$$



TABLE VI

Variations in the control matrix coefficients with variations in the pneumatic lead angle

HOVER	B <sub>11</sub>	B <sub>21</sub>	B <sub>31</sub>	B <sub>41</sub>	B <sub>51</sub>	B <sub>61</sub>	B <sub>71</sub>	B <sub>81</sub>
35°	-12.428	-8.730	9.730	0.0	3.662	-0.275	6.006	0.0
40°	-12.955	-8.697	9.356	0.0	1.164	-0.267	4.897	0.0
45°	-13.384	-8.597	9.370	0.0	-1.344	-0.256	3.75	0.0
35 KTS								
35°	-10.750	-9.950	7.543	0.0	0.857	-0.635	3.709	0.0
40°	-11.129	-6.778	7.565	0.0	-1.124	-0.643	2.816	0.0
45°	-11.423	-10.215	7.529	0.0	-3.097	-0.645	1.903	0.0
72 KTS								
35°	-8.260	-11.597	6.957	0.0	1.995	-1.171	3.637	0.0
40°	-8.474	-12.151	6.911	0.0	0.295	-1.237	2.842	0.0
45°	-8.624	-12.612	6.812	0.0	-1.407	-1.293	2.025	0.0
110 KTS								
35°	-7.508	-21.071	7.232	0.0	1.542	-1.167	3.961	0.0
40°	-7.593	-22.132	7.223	0.0	-0.053	-1.232	3.049	0.0
45°	-7.620	-23.024	7.158	0.0	-1.648	-1.288	2.114	0.0
130 KTS								
35°	-4.953	-26.501	6.733	0.0	1.613	-1.071	2.893	0.0
40°	-5.121	-27.611	6.726	0.0	0.714	-1.125	2.030	0.0
45°	-5.251	-28.511	6.674	0.0	-1.266	-1.170	1.151	0.0





showed that the variation of the pneumatic lead angle by plus or minus five (5) degrees had very little effect on the stability characteristics of the augmented plant matrix.



#### IV. CONCLUSIONS

A study has been made of the basic stability traits of the Kaman Aerospace Corporation XH-2/CCR helicopter, which is presently being constructed under NAVAIR contract as a technology demonstrator for the Circulation Control Rotor concept.

The airframe was defined by contractor generated stability and control derivatives which were then used to develop eigenvalues and eigenvectors for the system. The plant matrix (which characterizes the airframe) was generated by the MOSTAB program as modified to accommodate the CCR system, and the matrix coefficients represent the output from the program when it was operating in the 18 degree-of-freedom situation, i.e., six airframe degrees-of-freedom plus flapping, torsion and lead/lag degrees-of-freedom for the four blades ( $6 + 3 \times 4 = 18$ ). The uncompensated airframe characteristic roots at hover are in close accord with results obtained by the contractor for the six degree-of-freedom airframe, including a mild non-oscillatory instability ( $t_2 = 37.1$  sec.) that has been identified as due to a longitudinal short period type mode.

A feedback law has been identified using pitch attitude and pitch rate feedback into the longitudinal cyclic control that provides reasonable characteristic roots for the airframe. Presumably, further improvements could be obtained



by providing feedback in the lateral cyclic control system. The effect of varying the cyclic control lead angle on the pneumatic swashplate was investigated and found to be slight.

It should be noted that a unique feedback control law is not possible in modal control theory, when multiple control inputs (longitudinal and lateral cyclic) are available. Another way of stating this fact is that it is possible with several control inputs (and feedback laws) to have the same eigenvalues, but with different eigenvectors.

Finally, the characterization of the eigenvectors and identification of the eigenvalues with relevant modes was made possible by using a form of root locus analysis where the prime parameter was the amount of cross coupling. The trajectory of the characteristic roots as cross-coupling was linearly altered provided a physical insight into the history of the various roots.

Future studies are suggested to include estimating the mechanical gearing to the cockpit controls and then obtaining airframe response time histories for selected control inputs such as stick doublet type motions. Time histories can be generated quite readily using principles from control theory combined with calculated relations for system's transition matrices. It is quite possible that time history calculations for the compensated airframe will provide a better guide for selecting the control law. In any event, such studies will enhance satisfaction as to the question of airframe response behavior being reasonable.



# APPENDIX A

The basic plant matrix of the aircraft linearized equations of motion in the state variable format

THE COUPLED "A" MATRIX

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{p} \\ \dot{r} \\ \dot{v} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} X_u & X_w & X_q & -32.2 & X_p & X_r & X_v & 0.0 \\ Z_u & Z_w & U + Z_q & 0.0 & Z_p & Z_r & Z_v & 0.0 \\ M_u + Z_u M_w & M_w + m_w Z_w & M_q + M_w (U + Z_q) & 0.0 & M_p & M_r & M_v & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ L_u & L_w & L_q & 0.0 & L_p & L_r & L_v & 0.0 \\ N_u & N_w & N_q & 0.0 & N_p & N_r & N_v & 0.0 \\ Y_u & Y_w & Y_q & 0.0 & Y_p & Y_r - U & Y_v & 32.2 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ p \\ r \\ v \\ \phi \end{bmatrix}$$

U = Aircraft velocity in fps.





# APPENDIX B

## IBM-360 MODIFIED "BASMAT" PROGRAM

```

C BASIC MATRIX PROGRAM
C SUBPROGRAMS USED= CHREQ, SIMEQ, STAST, P330T, DET,
C CHREQA
0001 DIMENSION A(10,10),IGR(10),FIGI(10),C(11),AINV(10,10),
0002 1 NAME(5),J1(10,10),B(10,5),G(5,10)
0003 2001 FORMAT (5A4,3I2)
0004 2002 FORMAT (8E10,2I)
0005 2003 FORMAT (1P8E20,7I)
0006 2004 FORMAT (1HC,5X,16HTHE A MATRIX
0007 2005 FORMAT (1HC,5X,32HTHE CHARACTERISTIC POLYNOMIAL -
0008 * 24HTH ASCENDING POWERS OF S /)
0009 2006 FORMAT (1HC,5X,31HTHE EIGENVALUES OF THE A MATRIX)
0010 2007 FORMAT (8X,9HTHE REAL PART,8X,14HTHE IMAGINARY PART,/)
0011 2008 FORMAT (1H1,5X,20HTHE BASIC MATRIX PROGRAM)
0012 2009 FORMAT (6X,23HTHE PROBLEM IDENTIFICATION(ON=,5X,5A4)
0013 2010 FORMAT (1HC,5X,29HTHE DETERMINANT OF THE MATRIX/)
0014 2011 FORMAT (1HC,5X,25HTHE INVERSE OF THE MATRIX/)
0015 2012 FORMAT (1HC,45(1H*))
0016 2013 FORMAT (6I1)
0017 24 READ (5,2001) (NAME(I),I=1,5),N,M1,M2
0018 GO TO (3000,3001,7,10),M2
0019 DO 1 I=1,N
0020 C(I)=1.0
0021 DO 4 K=1,N
0022 A(I,K)=1(I,K)
0023 READ 2013, IDET,INV,NRM,ICP,IEIG,ISTM
0024 PRINT 2003
0025 PRINT 2005, (NAME(I),I=1,5)
0026 PRINT 2012
0027 PRINT 2004
0028 DO 2 I=1,N
0029 2 PRINT 2020, (A(I,K),K=1,N)
0030 IF (M1.EQ.0) GO TO 14
0031 PRINT 2012
0032 PRINT 2021
0033 DO 6 I=1,N
0034 3001 READ 2002, (B(I,K),K=1,M1)
0035 5 PRINT 2020, (B(I,K),K=1,M1)
0036 7 PRINT 2012
0037 PRINT 2022
0038 DO 3 I=1,M1
0039 READ 2002, (G(I,K),K=1,N)
0040 3 PRINT 2020, (G(I,K),K=1,N)
0041 PRINT 2023
0042 DO 13 I=1,N
0043 DO 12 K=1,N
0044 TEMP=0.
0045 DO 11 J=1,M1
0046 11 TEMP=TEMP+B(I,J)*G(J,K)
0047 12 A(I,K)=A(I,K)-TEMP
0048 13 PRINT 2020, (A(I,K),K=1,N)
0049 14 IF (IDET.NE.0) GO TO 5
0050 D=DET(A,N)
0051 PRINT 2010
0052 PRINT 2003, D
0053 5 IF (INV.NE.0) GO TO 15
0054 PRINT 2011
0055 CALL SIMEQ(A,C,N,AINV,C,TEMP)
0056 IF (ICP.EQ.0) GO TO 15
0057 DO 20 J=1,N
0058 20 PRINT 2003, (AINV(I,J),J=1,N)
0059 15 CALL CHREQ(A,A,C,N,M1)
0060 CALL P330T(N,C,IEIG,FIGI,*)
0061 IF (ICP.NE.0) GO TO 30
0062 PRINT 2012
0063 PRINT 2005
0064 NAME=N+1
0065 PRINT 2003, (C(I),I=1,NM)
0066 30 IF (IEIG.NE.0) GO TO 35
0067 PRINT 2012
0068 PRINT 2006
0069 PRINT 2007
0070 DO 3 I=1,N
0071 3 PRINT 2003, (IGR(I),FIGI(I))
0072 35 IF (ISTM.NE.0) GO TO 25
0073 CALL STAST(N,A,IEIG,FIGI,ISTM)
0074 25 GO TO 24
0075 11 PRINT
0076 2021 FORMAT (1P3E15,4)
0077 2021 FORMAT (1HC,5X,20HTHE CONTROL MATRIX B,/)
0078 2022 FORMAT (1HC,5X,17HTHE GAIN MATRIX G,/)
0079 2023 FORMAT (1HC,5X,31HTHE MODIFIED PLANT MATRIX, A-BG,/)
0080 END

```



```

0001      SUBROUTINE CHREQ(A,N,C,IRM)
0002      C THIS SUBROUTINE FINDS THE COEFFICIENTS OF THE CHARACTERISTIC
0003      C POLYNOMIAL USING THE LEVERIER ALGORITHM
0004      COMMON ZED(10,10,10)
0005      DIMENSION A(10,10),C(11),ATEMP(10,10),PRCD(10,10)
0006      DATA ATEMP/100*0.0/
0007      1000 FORMAT (1H0.5X,31H THE MATRIX COEFFICIENTS OF THE ,
0008      1001 33HNUMBER 1700 OF THE RESOLVENT MATRIX )
0009      1001 FORMAT (1H0.5X,29H THE MATRIX COEFFICIENT OF S**,11/)
0010      1002 FORMAT (1H0.45(1H*))
0011      CALL CHREQA(1,N,C)
0012      DO 55 I=1,N
0013      55 ATEMP(I,1)=1.0
0014      70 DO 30 J=1,N
0015      30 ZED(N,I,J)=ATEMP(I,J)
0016      IF (NRM.NE.0) GO TO 71
0017      WRITE (5,1003)
0018      WRITE (6,1000)
0019      M=N-1
0020      WRITE (6,1001) M
0021      DO 35 I=1,N
0022      35 WRITE (5,1002) (ATEMP(I,J),J=1,N)
0023      71 DO 40 I=1,N
0024      40 DO 40 J=1,N
0025      IF (I.EQ.1) GO TO 55
0026      IF (NRM.NE.0) GO TO 60
0027      WRITE (6,1001) NRM
0028      DO 45 J=1,N
0029      45 WRITE (5,1002) (ATEMP(J,K),K=1,N)
0030      60 NPM=N+1
0031      DO 90 I=1,N
0032      90 DO 20 J=1,N
0033      20 ZED(NP,I,J)=ATEMP(I,J)
0034      DO 15 J=1,N
0035      15 DO 15 K=1,N
0036      PRCD(J,K)=0.0
0037      DO 15 L=1,N
0038      15 ZED(J,K)=PRCD(J,K)+(A(J,L)*ATEMP(L,K))
0039      DO 13 J=1,N
0040      13 DO 13 K=1,N
0041      ATEMP(J,K)=PRCD(J,K)
0042      55 DO 10 J=1,N
0043      10 ATEMP(J,J)=ATEMP(J,J)+C(N-I+1)
0044      RETURN
0045      END
0001      SUBROUTINE CHREQA(1,N,C)
0002      DIMENSION J(11),C(11),B(10,10),A(10,10),D(300)
0003      M=N+1
0004      DO 20 I=1,N
0005      20 C(I)=0.0
0006      C(N)=1.0
0007      DO 14 M=1,N
0008      K=0
0009      L=1
0010      J(L)=1
0011      DO 1 J(L)=J(L)+1
0012      1 IF (J(L)=1) 3,5,50
0013      3 MM=M-1
0014      DO 4 I=L,M
0015      4 J(I)=J(I)+1
0016      DO 10 K=1,M
0017      NP=J(I)
0018      NC=J(KK)
0019      10 B(I,KK)=A(NR,NC)
0020      K=K+1
0021      C(I)=DET(A,M)
0022      DO 6 I=1,M
0023      6 I=I+1
0024      IF (J(I)-(M-M+L)) 1,3,50
0025      1 CONTINUE
0026      M=M-M+1
0027      DO 14 I=1,K
0028      14 C(M)=C(M)+D(I)*(-1.0)**M
0029      50 DET=0
0030      2000 FORMAT (1H0.5X,15H***** IN CHREQA)
0031      RETURN
0032      END

```







```

0001      SUBROUTINE STMT (N,A,EIGR,EIGI,KNOW)
0002      C THIS SUBROUTINE DETERMINES THE STATE TRANSITION MATRIX USING
0003      C SYLVESTER'S EXPANSION METHOD.
0004      DIMENSION CHI(10,10,10)
0005      DIMENSION A(10,10),EIGR(10),EIGI(10),SPS(10,10)
0006      COMPLEX CA(10,10),C1(10,10),C2(10,10),TCA(10,10),
0007      * DENOM(10),CEIG(10)
0008      1000 FORMAT (1H0,5X,25HTHE ELEMENTS OF THE STATE
0009      * 19H TRANSITION MATRIX X
0010      1001 FORMAT (1H0, 5X,25HTHE MATRIX COEFFICIENT OF
0011      * 5H EXP(,1PE13.6,7HTT*CS(,1PE13.6,2HTT/))
0012      1002 FORMAT (1P8F15.4)
0013      1003 FORMAT (1H0, 5X,25HTHE MATRIX COEFFICIENT OF
0014      * 5H EXP(,1PE13.6,7HTT*SIN(,1PE13.6,2HTT/))
0015      1004 FORMAT (1H0, 5X,25HTHE MATRIX COEFFICIENT OF
0016      * 5H EXP(,1PE13.6, 2HTT/))
0017      1005 FORMAT (1HC,45(1H*))
0018      IF (KNOW.NE.0) GO TO 800
0019      PRINT 1005
0020      DO 10 K=1,N
0021      CEIG(K)=CMPLX(EIGR(K),EIGI(K))
0022      DO 10 L=1,N
0023      CA(K,L) = CMPLX(A(K,L),0.0)
0024      IF (KNOW.NE.0) GO TO 700
0025      PRINT 1000
0026      DO 15 K=1,N
0027      CEIG(K)=CEIG(I)-CEIG(K)
0028      DO 500 J=1,N
0029      IF (J-I) 100,500,200
0030      IF (J-I) 110,110,150
0031      IF (I-I) 300,300,400
0032      IF (J-I-1) 110,110,150
0033      IF (J-I-1) 110,150,150
0034      DO 5 K=1,N
0035      DO 5 L=1,N
0036      C1(K,L)=CA(K,L)
0037      DO 20 K=1,N
0038      C1(K,K)=CA(K,K)-CEIG(J)
0039      DO 20 L=1,N
0040      C1(K,L)=CA(K,L)/DENOM(J)
0041      GO TO 500
0042      DO 40 K=1,N
0043      DO 40 L=1,N
0044      C2(K,L)=CA(K,L)
0045      DO 25 K=1,N
0046      C2(K,K)=CA(K,K)-CEIG(J)
0047      DO 25 L=1,N
0048      C2(K,L)=C2(K,L)/DENOM(J)
0049      DO 30 K=1,N
0050      DO 30 L=1,N
0051      TCA(K,L)=(C.0,C.0)
0052      DO 30 M=1,N
0053      TCA(K,L)=TCA(K,L) + C1(K,M)*C2(M,L)
0054      DO 35 K=1,N
0055      DO 35 L=1,N
0056      C1(K,L)=TCA(K,L)
0057      300 CONTINUE
0058      IF (KNOW.NE.0) GO TO 801
0059      PRINT 1001, EIGR(I),EIGI(I)
0060      DO 65 K=1,N
0061      DO 65 L=1,N
0062      SPS(K,L)=REAL(C1(K,L))*2.0
0063      DO 66 K=1,N
0064      DO 66 L=1,N
0065      SPS(K,L)=AIMAG(C1(K,L))*2.0
0066      600 CONTINUE
0067      IF (KNOW.NE.0) GO TO 802
0068      PRINT 1002, (SPS(K,L),L=1,N)
0069      PRINT 1003, EIGR(I),EIGI(I)
0070      DO 55 K=1,N
0071      DO 55 L=1,N
0072      SPS(K,L)=AIMAG(C1(K,L))*2.0
0073      DO 56 K=1,N
0074      DO 56 L=1,N
0075      SPS(K,L)=REAL(C1(K,L))*2.0

```





```

0072      DO 56 L=1,N
0073      CHI(I,K,L)=SPS(K,L)
0074      56 CONTINUE
0075      IF (IKNOW.NE.0) GO TO 600
0076      DO 35 K=1,N
0077      95 PRINT 1002, (SPS(K,L),L=1,N)
C ** CALCULATE COMPLEX EIGENVECTOR (1 MODIFICATION)
0078      210 PRINT 1007
0079      220 DO 221 K=1,N
0080      SPS(K,1)=SQRT(CHI(I,K,1)**2 + CHI(I,K,1)**2)
0081      SPS(K,2)=ATAN2(CHI(I,K,1),CHI(I,K,1)) *(-57.2958)
0082      PRINT 1007,SPS(K,1),SPS(K,2)
0083      221 CONTINUE
0084      1005 FORMAT(1H0.5X,26HTHE COMPLEX EIGENVECTOR IS,/3X,9HMAGNITUDE,
1 10X,11HPHASE (DEG)/)
0085      1007 FORMAT(1P2E20.7)
C ** END OF MODIFICATION.
0086      GO TO 600
0087      50 IF (IKNOW.NE.0) GO TO 304
0088      PRINT 1004, EIGR(I)
0089      304 DO 60 K=1,N
0090      DO 60 L=1,N
0091      60 SPS(K,L)=REAL (CAL(K,L))
0092      DO 61 K=1,N
0093      DO 61 L=1,N
0094      CHI(I,K,L)=SPS(K,L)
0095      61 CONTINUE
0096      IF (IKNOW.NE.0) GO TO 600
0097      DO 75 K=1,N
0098      75 PRINT 1002, (SPS(K,L),L=1,N)
0099      600 IF (I.GE.N) RETURN
0100      I=I+1
0101      GO TO 700
0102      END

```



```

0001      SUBROUTINE PROCT(N,A,U,V,I2)
C      THIS SUBROUTINE USES A MODIFIED BARSTON METHOD TO FIND THE
C      ROOTS OF A POLYNOMIAL.
0002      DIMENSION A(20),U(20),V(20),H(21),B(21),C(21)
0003      IREV=IR
0004      NC=N+1
0005      DO11=1,NC
0006      1 H(I)=A(I)
0007      P=C.
0008      Q=C.
0009      R=0.
0010      3 IF(H(1))4,2,4
0011      2 NC=NC-1
0012      V(NC)=0.
0013      U(NC)=0.
0014      C11002=1,NC
0015      1002 H(I)=H(I+1)
0016      GOT13
0017      4 IF(NC-1)5,100,5
0018      5 IF(NC-2)7,6,7
0019      6 R=-H(1)/H(2)
0020      GOT150
0021      7 IF(NC-3)6,6,6
0022      8 P=H(2)/H(3)
0023      Q=H(1)/H(3)
0024      GOT170
0025      9 IF(ABS (H(NC-1)/H(NC))-ABS (H(2)/H(1)))10,19,19
0026      10 IREV=-IREV
0027      M=NC/2
0028      DO111=1,M
0029      NL=NC+1-I
0030      R=H(NL)
0031      H(NL)=H(I)
0032      11 H(I)=R
0033      IF(0)13,12,13
0034      12 P=0.
0035      GOT15
0036      13 P=P/Q
0037      Q=1./Q
0038      15 IF(2)16,19,16
0039      16 R=1./Q
0040      19 R=5./5-10
0041      R(NC)=H(NC)
0042      C(NC)=H(NC)
0043      R(NC+1)=0.
0044      C(NC+1)=0.
0045      NP=NC-1
0046      20 C1499=1,1000
0047      DO2111=1,NP
0048      I=NC-1
0049      B(I)=H(I)+R*B(I+1)
0050      21 C(I)=B(I)+P*C(I+1)
0051      IF(ABS (B(1)/H(1))-150,50,24
0052      24 IF(C(2))23,22,23
0053      22 R=P+1.
0054      GOT130

```



```

0055      23 R=R-B(1)/C(2)
0056      32 DO 27 I=1,NP
0057          Y=NC-I
0058          B(I)=H(I)-F*B(I+1)-Q*B(I+2)
0059      37 C(I)=B(I)-B*(I+1)-Q*C(I+2)
0060      JJ6J      IF(H(Z))32,31,32
0061      31 IF(ABS (B(2)/H(1)))-E)33,33,34
0062      32 IF(ABS (B(2)/H(2)))-E)33,33,34
0063      33 IF(ABS (B(1)/H(1)))-E)70,70,34
0064      34 CBAP=C(2)-B(2)
0065          Q=C(3)*2-C(4)*C(4)
0066          IF(D)36,35,36
0067      35 Q=2-2
0068          Q=Q*(Q+1.)
0069          GOT749
0070      36 Q=B*(2)*C(3)-B(1)*C(4))/D
0071          Q=C+(-B(2)*CBAP+B(1)*C(3))/D
0072      49 CONTINUE
0073          F=E*10.
0074          GOT720
0075      50 NC=NC-1
0076          V(NC)=0.
0077          IF(IPEV)51,52,52
0078      51 U(NC)=1./2
0079          GOT753
0080      52 U(NC)=R
0081      53 DO 54 I=1,NC
0082      54 H(I)=B(I+1)
0083          GOT74
0084      70 NC=NC-2
0085          IF(IPEV)71,72,72
0086      71 QP=1./Q
0087          PP=P/(Q*2.C)
0088          GOT773
0089      72 CP=0
0090          PP=P/2.0
0091      73 F=(PP)*2-Q2
0092          IF(F)4,75,75
0093      74 U(NC+1)=-PP
0094          U(NC)=-PP
0095          V(NC+1)=SQRT (-F)
0096          V(NC)=-V(NC+1)
0097          GOT776
0098      75 IF(PP)81,80,81
0099      80 U(NC+1)=-SQRT(F)
0100          GO TO 82
0101      81 U(NC+1)=-((PP/ABS (PP))*(ABS (PP)+SQRT (F))
0102      82 CONTINUE
0103          V(NC+1)=0.
0104          U(NC)=QP/U(NC+1)
0105          V(NC)=0.
0106      76 DO 77 I=1,NC
0107      77 H(I)=B(I+2)
0108          GOT74
0109      100 RETURN
0110      END

```



# APPENDIX C

## MODIFIED "BASMAT" PROGRAM FOR HP-9830

1	CIM A(6,8),C(8,8)	BAS00010
2	CIM M(8,1),T(1,8)	BAS00020
4	CIM Z(8,8)	BAS00030
6	FIXED 6	BAS00040
7	DATA	BAS00050
8	DATA	BAS00060
9	DATA	BAS00070
10	DATA	BAS00080
11	DATA	BAS00090
12	DATA	BAS00100
13	DATA	BAS00110
14	DATA	BAS00120
15	DATA	BAS00130
16	MAT READ O	BAS00140
17	PRINT "CCR 130 KNOTS THE BASIC A MATRIX"	BAS00150
18	MAT PRINT C	BAS00160
19	PRINT	BAS00170
20	MAT READ M	BAS00180
21	PRINT "THE BASIC CONTRCL MATRIX IS"	BAS00190
22	MAT PRINT M	BAS00200
23	FOR I5=-1 TO 1 STEP C.1	BAS00210
24	FOR J5=-1 TO 1 STEP C.1	BAS00220
25	Y(1,1)=Y(1,2)=Y(1,5)=Y(1,6)=Y(1,7)=Y(1,8)=0	BAS00230
27	Y(1,3)=I5	BAS00240
28	Y(1,4)=J5	BAS00250
29	MAT Z=M*Y	BAS00260
30	MAT A=C-Z	BAS00270
31	G1=C	BAS00280
32	GCSUB 2000	BAS00290
33	N=8	BAS00300
34	PRINT "THE REVISED PLANT MATRIX AT 130 KTS, WITH K(Q1)=",I5	BAS00310
35	PRINT "AND K(THETA)=",J5	BAS00320
36	MAT PRINT A	BAS00330
37	REDIM K(8,8)	BAS00340
38	PRINT	BAS00350
43	FOR I=1 TO N	BAS00360
44	F(1,1)=M(1,1)	BAS00370
46	NEXT I	BAS00380
48	FOR I=2 TO N	BAS00390
50	L5=I-1	BAS00400
52	FOR J=1 TO N	BAS00410
54	F(J,I)=0	BAS00420
56	FOR K=1 TO N	BAS00430
58	F(J,I)=F(J,I)+A(J,K)*F(K,L5)	BAS00440
60	NEXT K	BAS00450
61	NEXT J	BAS00460
62	NEXT I	BAS00470
63	FOR I=1 TO N	BAS00480
64	FOR J=1 TO N	BAS00490
65	K(I,J)=A(I,J)	BAS00500
66	A(I,J)=F(I,J)	BAS00510
67	NEXT J	BAS00520
68	NEXT I	BAS00530
69	GCSUB 1000	BAS00540
70	IF E NOT EQUAL TO ZERO THEN 76	BAS00550
71	GCSUB 2000	BAS00560
72	PRINT "MATRIX IS UNCONTROLLABLE"	BAS00570
74	GCTC 110	BAS00580
76	FOR I=1 TO N	BAS00590
78	FOR J=1 TO N	BAS00600
79	Q(I,J)=C	BAS00610
80	FOR K=1 TO N	BAS00620
81	Q(I,J)=Q(I,J)+F(I,K)*P(K,J)	BAS00630
82	NEXT K	BAS00640
82	NEXT K	BAS00650
84	NEXT J	BAS00660
86	NEXT I	BAS00670
88	E1=C	BAS00680
90	FOR I=1 TO N	BAS00690
92	FOR J=1 TO N	BAS00700
93	A(I,J)=K(I,J)	BAS00710
94	IF (I=J) NOT EQUAL TO ZERO THEN 100	BAS00720
96	E2=ABS(C(I,J)-1)	BAS00730
98	GOTO 101	BAS00740
100	E2=ABS(C(I,J))	BAS00750
101	IF ABS(E1)>ABS(E2) THEN 104	BAS00760
102	E1=ABS(E2)	BAS00770
103	GCTC 105	BAS00780
104	E1=ABS(E1)	BAS00790
105	NEXT J	BAS00800
106	NEXT I	BAS00810
107	PRINT	BAS00820
108	IF (E1-E-C5)<0 THEN 110	BAS00830
109	PRINT "PLANT IS NUMERICALLY UNCONTROLLABLE, DEVIATION=";E1	BAS00840
110	GCSUB 2000	BAS00850
112	PRINT "OPEN LOOP CALCULATIONS"	BAS00860
114	PRINT	BAS00870
116	PRINT "CENEX COEF IN ASCENDING POWERS OF S"	BAS00880





120	GOSUB 3199	BASC00890
122	FOR I=1 TO N6	BASC00900
124	PRINT C(I)	BASC00910
126	L(I)=C(I)	BASC00920
128	NEXT I	BASC00930
130	GOSUB 3399	BASC00940
132	GOSUB 5000	BASC00950
430	NEXT J5	BASC00960
440	NEXT I5	BASC00970
450	END	BASC00980
1000	N7=1	BASC00990
1002	E=1	BASC01000
1004	FOR I=1 TO N	BASC01010
1006	FOR J=1 TO N	BASC01020
1008	P(I,J)=0	BASC01030
1010	G(I,J)=A(I,J)	BASC01040
1012	NEXT J	BASC01050
1014	NEXT I	BASC01060
1016	FOR I=1 TO N	BASC01070
1018	X(I)=1	BASC01080
1019	P(I,I)=1	BASC01090
1020	NEXT I	BASC01100
1022	FOR I=1 TO N	BASC01110
1024	C=C	BASC01120
1026	K=1	BASC01130
1028	IF (ABS(G(K,I))-ABS(C)) <= 0 THEN 1034	BASC01140
1030	C=G(K,I)	BASC01150
1032	N7=K	BASC01160
1034	K=K+1	BASC01170
1036	IF (K-N) <= 0 THEN 1028	BASC01180
1038	IF (G(N7,I))=0 THEN 1102	BASC01190
1040	IF (N7-I)<0 THEN 1102	BASC01200
1042	IF (N7-I)=0 THEN 1066	BASC01210
1044	FOR M=1 TO N	BASC01220
1046	T=G(I,M)	BASC01230
1048	G(I,M)=G(N7,M)	BASC01240
1050	G(N7,M)=T	BASC01250
1052	T=P(I,M)	BASC01260
1054	P(I,M)=P(N7,M)	BASC01270
1056	P(N7,M)=T	BASC01280
1058	NEXT M	BASC01290
1060	T=X(I)	BASC01300
1062	X(I)=X(N7)	BASC01310
1064	X(N7)=T	BASC01320
1066	X(I)=X(I)/G(I,I)	BASC01330
1068	T=X(I)	BASC01340
1068	T=G(I,I)	BASC01350
1070	FOR M=1 TO N	BASC01360
1072	P(I,M)=P(I,M)/T	BASC01370
1074	G(I,M)=G(I,M)/T	BASC01380
1076	NEXT M	BASC01390
1078	FOR J=1 TO N	BASC01400
1080	IF (J-I)=0 THEN 1096	BASC01410
1082	IF (G(J,I))=0 THEN 1096	BASC01420
1084	X(J)=X(J)-G(J,I)*X(I)	BASC01430
1086	T=G(J,I)	BASC01440
1088	FOR N7=1 TO M	BASC01450
1090	P(J,N7)=P(J,N7)-T*P(I,N7)	BASC01460
1092	G(J,N7)=G(J,N7)-T*G(I,N7)	BASC01470
1094	NEXT N7	BASC01480
1096	NEXT J	BASC01490
1098	NEXT I	BASC01500
1100	RETURN	BASC01510
1102	PRINT "MATRIX IS SINGULAR"	BASC01520
1104	E=C	BASC01530
1106	RETURN	BASC01540
1500	FOR I=1 TO N6	BASC01550
1502	R(I)=C(I)	BASC01560
1504	NEXT I	BASC01570
1506	RETURN	BASC01580
2000	PRINT "*****"	BASC01590
2004	RETURN	BASC01600
3199	N6=N+1	BASC01610
3200	CIM C(100)	BASC01620
3201	FOR I=1 TO N6	BASC01630
3203	C(I)=0	BASC01640
3205	NEXT I	BASC01650
3207	C(N6)=1	BASC01660
3209	FOR M=1 TO N	BASC01670
3211	A=C	BASC01680
3213	L=1	BASC01690
3215	J(L)=1	BASC01700
3217	GOTO 3221	BASC01710
3219	J(L)=J(L)+1	BASC01720
3221	IF (L-M)=0 THEN 3235	BASC01730
3223	IF (L-M)>0 THEN 3277	BASC01740
3225	M=M-1	BASC01750
3227	FOR I=L TO M1	BASC01760
3229	II=I+1	BASC01770



3231	J(I1)=J(I)+1	BASC1780
3233	NEXT I	BASC1790
3235	FOR I=1 TO M	BASC1800
3237	FOR K1=1 TO M	BASC1810
3239	N5=J(I)	BASC1820
3241	N1=J(K1)	BASC1830
3243	B(I,K1)=A(N5,N1)	BASC1840
3245	NEXT K1	BASC1850
3247	NEXT I	BASC1860
3249	A=A+1	BASC1870
3251	GCSUB 3299	BASC1880
3253	C(A)=0	BASC1890
3255	FOR I=1 TO M	BASC1900
3257	L=M-I+1	BASC1910
3259	IF (J(L)-(A-M+L))<C THEN 3219	BASC1920
3261	IF (J(L)-(A-M+L))>C THEN 3277	BASC1930
3263	NEXT I	BASC1940
3265	M2=A-M+1	BASC1950
3267	FOR I=1 TO A	BASC1960
3269	C(M2)=C(M2)+C(I)*(-1)EXP(M)	BASC1970
3271	NEXT I	BASC1980
3273	NEXT M	BASC1990
3275	RETURN	BASC2000
3277	PRINT " ERROR IN CFREQ"	BASC2010
3279	RETURN	BASC2020
3259	I3=C	BASC2030
3301	FOR I=1 TO M	BASC2040
3303	K=I	BASC2050
3305	IF (B(K,I)) NOT EQUAL TO ZERO THEN 3313	BASC2060
3307	C THEN 3313	BASC2070
3309	K=K+1	BASC2080
3311	IF (K-M) <= 0 THEN 3305	BASC2090
3313	IF (K-M) > 0 THEN 3359	BASC2100
3315	IF (I-K) = 0 THEN 3329	BASC2110
3317	IF (I-K) > 0 THEN 3359	BASC2120
3319	FOR M4=1 TO M	BASC2130
3321	T=B(I,M4)	BASC2140
3323	B(K,M4)=T	BASC2150
3325	NEXT M4	BASC2160
3327	I3=I3+1	BASC2170
3329	I1=I+1	BASC2180
3331	IF I1>M THEN 3345	BASC2190
3333	FOR M4=I1 TO M	BASC2200
3335	IF B(M4,I)=C THEN 3344	BASC2210
3337	T=B(M4,I)/B(I,I)	BASC2220
3339	FOR B=1 TO M	BASC2230
3341	B(M4,B)=B(M4,B)-B(I,B)*T	BASC2240
3343	NEXT B	BASC2250
3345	NEXT M4	BASC2260
3347	D=1	BASC2270
3349	FOR I=1 TO M	BASC2280
3351	C=C*B(I,1)	BASC2290
3353	NEXT I	BASC2300
3355	C=((I-1)*EXP(I3)*D	BASC2310
3357	RETURN	BASC2320
3359	D=C	BASC2330
3361	RETURN	BASC2340
3363	I3=1	BASC2350
3401	N1=N+1	BASC2360
3403	FOR I=1 TO N1	BASC2370
3405	D(I)=C(I)	BASC2380
3407	NEXT I	BASC2390
3409	P=Q=R=0	BASC2400
3411	IF D(1) NOT EQUAL TO ZERO THEN 3425	BASC2410
3413	0 THEN 3425	BASC2420
3415	N1=N1-1	BASC2430
3417	V(N1)=U(N1)=C	BASC2440
3419	FOR I=1 TO N1	BASC2450
3421	D(I)=D(I+1)	BASC2460
3423	NEXT I	BASC2470
3425	GCTC 3411	BASC2480
3427	IF (N1-1)=C THEN 3615	BASC2490
3429	IF (N1-2) NOT EQUAL TO ZERO THEN 3433	BASC2500
3431	R=C(1)/C(2)	BASC2510
3433	GCTC 3549	BASC2520
3435	IF (N1-3) NOT EQUAL TO ZERO THEN 3441	BASC2530
3437	P=O(2)/C(3)	BASC2540
3439	Q=O(1)/C(3)	BASC2550
3441	GCTC 3567	BASC2560
3443	IF (ABS(C(N1-10)/D(N1)-ABS(C(2)/O(1))) >= 0 THEN 3437	BASC2570
3445	I3=-I3	BASC2580
3447	M=N1/2	BASC2590
3449	FOR I=1 TO M	BASC2600
3451	N2=N1+1-I	BASC2610
3453	F=C(N2)	BASC2620
3455	C(N2)=C(I)	BASC2630
3457	C(N2)=C(I)	BASC2640
3459	C(N2)=C(I)	BASC2650



3455	C(I)=F	BASC02660
3457	NEXT I	BASC02670
3459	IF C NOT EQUAL TO ZERO THEN 3465	BASC02680
3461	P=C	BASC02690
3463	GC TC 3469	BASC02700
3465	F=P/C	BASC02710
3467	C=1/C	BASC02720
3469	IF R=C THEN 3473	BASC02730
3471	R=1/R	BASC02740
3473	E=5E-1C	BASC02750
3475	J(N1)=E(N1)	BASC02760
3477	W(N1)=C(N1)	BASC02770
3479	J(N1+1)=W(N1+1)=0	BASC02780
3483	N3=N1-1	BASC02790
3485	FCR I=1 TC 1000	BASC02800
3487	FCR I1=1 TC N3	BASC02810
3489	I=N1-1	BASC02820
3491	J(I)=C(I)+R*J(I+1)	BASC02830
3493	W(I)=J(I)+R*W(I+1)	BASC02840
3495	NEXT I	BASC02850
3497	IF (ABS(J(1)/D(1)-E) <= 0 THEN 3549	BASC02860
3499	IF W(2) NOT EQUAL TO ZERO THEN 3505	BASC02870
3501	R=R+1	BASC02880
3503	GC TC 3507	BASC02890
3505	R=R-J(I)/W(2)	BASC02900
3507	FCR I1=1 TC N3	BASC02910
3509	I=N1-1	BASC02920
3511	J(I)=C(I)-F*J(I+1)-C*J(I+2)	BASC02930
3513	W(I)=J(I)-F*W(I+1)-C*W(I+2)	BASC02940
3515	NEXT I	BASC02950
3517	IF C(2) NOT EQUAL TO ZERO THEN 3523	BASC02960
3519	IF (ABS(J(2)/D(1))-E) > THEN 3527	BASC02970
3521	IF (ABS(J(2)/D(1))-E) <= THEN 3525	BASC02980
3523	IF (ABS(J(2)/D(2))-E) > 0 THEN 3527	BASC02990
3525	IF (ABS(J(1)/D(1))-E) <= 0 THEN 3567	BASC03000
3527	C1=W(2)-J(2)	BASC03010
3529	D=W(3)*EXP(2)-C1*W(4)	BASC03020
3531	IF D NOT EQUAL TO ZERO THEN 3539	BASC03030
3533	P=P-2	BASC03040
3535	C=C*(C+1)	BASC03050
3537	GC TC 3543	BASC03060
3539	P=P+(J(2)*W(3)-J(1)*W(4))/C	BASC03070
3541	Q=Q+(-J(2)*C1+J(1)*W(3))/D	BASC03080
3543	NEXT I	BASC03090
3545	E=E*10	BASC03100
3547	GC TC 3485	BASC03110
3549	N1=N1-1	BASC03120
3551	V(N1)=0	BASC03130
3553	IF I3 >= C THEN 3559	BASC03140
3555	U(N1)=1/R	BASC03150
3557	GC TC 3561	BASC03160
3559	U(N1)=R	BASC03170
3561	FCR I=1 TC N1	BASC03180
3563	D(I)=J(I+1)	BASC03190
3565	NEXT I	BASC03200
3567	GC TC 3425	BASC03210
3569	N1=N1-2	BASC03220
3571	IF I3 >= 0 THEN 3577	BASC03230
3573	Q1=1/C	BASC03240
3575	P1=P/TC*2	BASC03250
3577	GC TC 3581	BASC03260
3579	Q1=C	BASC03270
3581	P1=P/2	BASC03280
3583	F=P1*EXP(2)-Q1	BASC03290
3585	IF F >= 0 THEN 3593	BASC03300
3587	U(N1+1)=C(N1)=-P1	BASC03310
3589	V(N1+1)=SQR(-F)	BASC03320
3591	V(N1)=-V(N1+1)	BASC03330
3593	GC TC 3607	BASC03340
3595	IF P1 NOT EQUAL TO ZERO THEN 3599	BASC03350
3597	U(N1+1)=-SQR(F)	BASC03360
3599	GC TC 3601	BASC03370
3601	U(N1+1)=-(P1/ABS(P1))*(ABS(P1)+SQR(F))	BASC03380
3603	V(N1+1)=V(N1)=0	BASC03390
3605	U(N1)=C1/U(N1+1)	BASC03400
3607	FCR I=1 TC N1	BASC03410
3609	C(I)=J(I+2)	BASC03420
3611	NEXT I	BASC03430
3613	GC TC 3425	BASC03440
3615	RETURN	BASC03450
3617	RETURN	BASC03460
3619	PRINT	BASC03470
3621	PRINT " THE ROOTS ARE: REAL INAGINARY"	BASC03480
3623	FCR I=1 TC N	BASC03490
3625	PRINT " ;U(I),V(I)	BASC03500
3627	NEXT I	BASC03510
3629	PRINT	BASC03520
3631	RETURN	BASC03530





# APPENDIX D Sample BASMAT computer program output.

BASIC MATRIX PROGRAM  
 PPTLE= IDENTIFICATION- POWER 40 CFC 1.0

\*\*\*\*\*

## THE A MATRIX

-1.73900-03	-1.44300-03	2.33500 CC	-3.22000 01	7.69300-01	-2.73500-01	2.23100-02	0.0
-5.03300-03	-2.36300-01	-1.41700-01	0.0	6.72000-01	6.57400-01	-1.73800-02	0.0
8.29270-04	-3.19300-04	-2.54200 03	0.0	-1.98100 00	1.68800-01	-1.03400-02	0.0
0.0	0.0	1.00000 CC	0.0	0.0	0.0	0.0	0.0
-4.39900-03	-1.70600-03	6.45900 00	0.0	-7.71300 00	-2.74800-01	-1.21900-02	0.0
3.52000-05	-1.90000-03	1.31500-01	0.0	-2.04500-01	-3.66100-01	1.25700-02	0.0
-2.64000-03	-1.42400-02	7.25100-01	0.0	-2.47100 00	5.60800-01	-2.60500-02	3.22000 01
0.0	0.0	0.0	0.0	1.00000 00	0.0	0.0	0.0

\*\*\*\*\*

## THE CONTROL MATRIX E

-1.29550 01
-8.69700 00
9.36600 00
0.0
1.16400 00
-2.67000-01
4.85700 00
0.0

\*\*\*\*\*

## THE GAIN MATRIX E

0.0	0.0	0.0	0.0	0.0	0.0	0.0
-----	-----	-----	-----	-----	-----	-----

## THE MODIFIED PLANT MATRIX, A-PC

-1.76900-03	-1.44300-03	2.33500 00	-3.22000 01	7.69300-01	-2.73500-01	2.23100-02	0.0
-5.03300-03	-2.36300-01	-1.41700-01	0.0	6.72000-01	6.57400-01	-1.73800-02	0.0
8.29270-04	-3.19300-04	-2.54200 00	0.0	-1.98100 00	1.68800-01	-1.03400-02	0.0
0.0	0.0	1.00000 00	0.0	0.0	0.0	0.0	0.0
-4.39900-03	-1.70600-03	6.45900 00	0.0	-7.71300 00	-2.74800-01	-1.21900-02	0.0
3.52000-05	-1.90000-03	1.31500-01	0.0	-2.04500-01	-3.66100-01	1.25700-02	0.0
-2.64000-03	-1.42400-02	7.25100-01	0.0	-2.47100 00	5.60800-01	-2.60500-02	3.22000 01
0.0	0.0	0.0	0.0	1.00000 00	0.0	0.0	0.0

\*\*\*\*\*

## THE CHARACTERISTIC POLYNOMIAL - IN ASCENDING POWERS OF S

-1.12717830-04	3.30094230-03	1.41502950-01	1.02599150 00	5.27256770 00	2.19051210 01
3.98056090 01	1.05857400 01	1.00000000 00			





THE EIGENVALUES OF THE MARGINAL  
REAL PART IMAGINARY PART

0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3.0 3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 4.0 4.1 4.2 4.3 4.4 4.5 4.6 4.7 4.8 4.9 5.0 5.1 5.2 5.3 5.4 5.5 5.6 5.7 5.8 5.9 6.0 6.1 6.2 6.3 6.4 6.5 6.6 6.7 6.8 6.9 7.0 7.1 7.2 7.3 7.4 7.5 7.6 7.7 7.8 7.9 8.0 8.1 8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 9.0 9.1 9.2 9.3 9.4 9.5 9.6 9.7 9.8 9.9 1.00 1.01 1.02 1.03 1.04 1.05 1.06 1.07 1.08 1.09 1.10 1.11 1.12 1.13 1.14 1.15 1.16 1.17 1.18 1.19 1.20 1.21 1.22 1.23 1.24 1.25 1.26 1.27 1.28 1.29 1.30 1.31 1.32 1.33 1.34 1.35 1.36 1.37 1.38 1.39 1.40 1.41 1.42 1.43 1.44 1.45 1.46 1.47 1.48 1.49 1.50 1.51 1.52 1.53 1.54 1.55 1.56 1.57 1.58 1.59 1.60 1.61 1.62 1.63 1.64 1.65 1.66 1.67 1.68 1.69 1.70 1.71 1.72 1.73 1.74 1.75 1.76 1.77 1.78 1.79 1.80 1.81 1.82 1.83 1.84 1.85 1.86 1.87 1.88 1.89 1.90 1.91 1.92 1.93 1.94 1.95 1.96 1.97 1.98 1.99 2.00 2.01 2.02 2.03 2.04 2.05 2.06 2.07 2.08 2.09 2.10 2.11 2.12 2.13 2.14 2.15 2.16 2.17 2.18 2.19 2.20 2.21 2.22 2.23 2.24 2.25 2.26 2.27 2.28 2.29 2.30 2.31 2.32 2.33 2.34 2.35 2.36 2.37 2.38 2.39 2.40 2.41 2.42 2.43 2.44 2.45 2.46 2.47 2.48 2.49 2.50 2.51 2.52 2.53 2.54 2.55 2.56 2.57 2.58 2.59 2.60 2.61 2.62 2.63 2.64 2.65 2.66 2.67 2.68 2.69 2.70 2.71 2.72 2.73 2.74 2.75 2.76 2.77 2.78 2.79 2.80 2.81 2.82 2.83 2.84 2.85 2.86 2.87 2.88 2.89 2.90 2.91 2.92 2.93 2.94 2.95 2.96 2.97 2.98 2.99 3.00 3.01 3.02 3.03 3.04 3.05 3.06 3.07 3.08 3.09 3.10 3.11 3.12 3.13 3.14 3.15 3.16 3.17 3.18 3.19 3.20 3.21 3.22 3.23 3.24 3.25 3.26 3.27 3.28 3.29 3.30 3.31 3.32 3.33 3.34 3.35 3.36 3.37 3.38 3.39 3.40 3.41 3.42 3.43 3.44 3.45 3.46 3.47 3.48 3.49 3.50 3.51 3.52 3.53 3.54 3.55 3.56 3.57 3.58 3.59 3.60 3.61 3.62 3.63 3.64 3.65 3.66 3.67 3.68 3.69 3.70 3.71 3.72 3.73 3.74 3.75 3.76 3.77 3.78 3.79 3.80 3.81 3.82 3.83 3.84 3.85 3.86 3.87 3.88 3.89 3.90 3.91 3.92 3.93 3.94 3.95 3.96 3.97 3.98 3.99 4.00 4.01 4.02 4.03 4.04 4.05 4.06 4.07 4.08 4.09 4.10 4.11 4.12 4.13 4.14 4.15 4.16 4.17 4.18 4.19 4.20 4.21 4.22 4.23 4.24 4.25 4.26 4.27 4.28 4.29 4.30 4.31 4.32 4.33 4.34 4.35 4.36 4.37 4.38 4.39 4.40 4.41 4.42 4.43 4.44 4.45 4.46 4.47 4.48 4.49 4.50 4.51 4.52 4.53 4.54 4.55 4.56 4.57 4.58 4.59 4.60 4.61 4.62 4.63 4.64 4.65 4.66 4.67 4.68 4.69 4.70 4.71 4.72 4.73 4.74 4.75 4.76 4.77 4.78 4.79 4.80 4.81 4.82 4.83 4.84 4.85 4.86 4.87 4.88 4.89 4.90 4.91 4.92 4.93 4.94 4.95 4.96 4.97 4.98 4.99 5.00 5.01 5.02 5.03 5.04 5.05 5.06 5.07 5.08 5.09 5.10 5.11 5.12 5.13 5.14 5.15 5.16 5.17 5.18 5.19 5.20 5.21 5.22 5.23 5.24 5.25 5.26 5.27 5.28 5.29 5.30 5.31 5.32 5.33 5.34 5.35 5.36 5.37 5.38 5.39 5.40 5.41 5.42 5.43 5.44 5.45 5.46 5.47 5.48 5.49 5.50 5.51 5.52 5.53 5.54 5.55 5.56 5.57 5.58 5.59 5.60 5.61 5.62 5.63 5.64 5.65 5.66 5.67 5.68 5.69 5.70 5.71 5.72 5.73 5.74 5.75 5.76 5.77 5.78 5.79 5.80 5.81 5.82 5.83 5.84 5.85 5.86 5.87 5.88 5.89 5.90 5.91 5.92 5.93 5.94 5.95 5.96 5.97 5.98 5.99 6.00 6.01 6.02 6.03 6.04 6.05 6.06 6.07 6.08 6.09 6.10 6.11 6.12 6.13 6.14 6.15 6.16 6.17 6.18 6.19 6.20 6.21 6.22 6.23 6.24 6.25 6.26 6.27 6.28 6.29 6.30 6.31 6.32 6.33 6.34 6.35 6.36 6.37 6.38 6.39 6.40 6.41 6.42 6.43 6.44 6.45 6.46 6.47 6.48 6.49 6.50 6.51 6.52 6.53 6.54 6.55 6.56 6.57 6.58 6.59 6.60 6.61 6.62 6.63 6.64 6.65 6.66 6.67 6.68 6.69 6.70 6.71 6.72 6.73 6.74 6.75 6.76 6.77 6.78 6.79 6.80 6.81 6.82 6.83 6.84 6.85 6.86 6.87 6.88 6.89 6.90 6.91 6.92 6.93 6.94 6.95 6.96 6.97 6.98 6.99 7.00 7.01 7.02 7.03 7.04 7.05 7.06 7.07 7.08 7.09 7.10 7.11 7.12 7.13 7.14 7.15 7.16 7.17 7.18 7.19 7.20 7.21 7.22 7.23 7.24 7.25 7.26 7.27 7.28 7.29 7.30 7.31 7.32 7.33 7.34 7.35 7.36 7.37 7.38 7.39 7.40 7.41 7.42 7.43 7.44 7.45 7.46 7.47 7.48 7.49 7.50 7.51 7.52 7.53 7.54 7.55 7.56 7.57 7.58 7.59 7.60 7.61 7.62 7.63 7.64 7.65 7.66 7.67 7.68 7.69 7.70 7.71 7.72 7.73 7.74 7.75 7.76 7.77 7.78 7.79 7.80 7.81 7.82 7.83 7.84 7.85 7.86 7.87 7.88 7.89 7.90 7.91 7.92 7.93 7.94 7.95 7.96 7.97 7.98 7.99 8.00 8.01 8.02 8.03 8.04 8.05 8.06 8.07 8.08 8.09 8.10 8.11 8.12 8.13 8.14 8.15 8.16 8.17 8.18 8.19 8.20 8.21 8.22 8.23 8.24 8.25 8.26 8.27 8.28 8.29 8.30 8.31 8.32 8.33 8.34 8.35 8.36 8.37 8.38 8.39 8.

# THE ELEMENTS OF THE CALCULUS

THE MIX (CEFF) - 1.8607(0)-C21Y

[illegible]

THE MATRIX EFFECTS OF FEXF (-5, 1895COP-C21Y

4. 37720-01	-1. 75250-03	2. 71450-02	3. 52620-02	6. 02070-01	8. 61820-01	-1. 57600-00	5. 48250-02
4. 37730-02	-1. 74230-00	-5. 72430-00	-1. 08420-01	-2. 15700-00	-3. 08760-00	-3. 56470-02	-3. 56470-02
1. 1. 05430-05	-1. 49530-00	-2. 27730-02	-2. 52100-02	-4. 94320-03	-7. 17100-03	-1. 21510-05	-1. 14950-00
7. 85910-04	-2. 87820-00	-2. 55770-01	-5. 85790-01	-5. 66340-02	-1. 34350-01	-2. 57350-02	-1. 57340-00
5. 53920-03	3. 10100-05	-1. 05050-01	-5. 36700-03	-1. 04350-03	-1. 51490-01	-2. 80930-02	-1. 7. 3300-00
7. 7430-03	3. 10100-05	-2. 06730-03	-2. 32640-03	-4. 62840-01	-6. 2510-01	-1. 21200-02	-7. 3200-00
7. 1590-02	1. 57530-01	-5. 47500-01	-2. 02850-01	-1. 19540-01	-1. 71480-01	-3. 14000-01	-1. 5480-02
6. 6560-02	-6. 12750-03	-5. 28300-02	-1. 03440-01	-2. 05620-02	-2. 54540-02	-3. 33920-01	-3. 34370-01

THE MATRIX COEFFICIENTS OF THE EXP(-6.3446471) C(3)FC-S(2.9043ED-01)T

3.24460-01	1.14250-01	-1.71910	1.32830	01	-9.22920	00	1.17770	03	1.5515	00	-5.23430	CC
4.47740-03	-1.14940-03	-1.59240-02	1.52330	-02	5.65460-02	-1.64890-02	-1.64890-02	03	2.63715	-03	-1.01550	CC
5.47610-04	1.45340-04	-2.04500-01	3.25430	-02	-1.69440-01	1.69440-01	1.69440-01	03	-3.56425	-03	-1.14770-01	CC
0.00690-04	0.54540-04	-2.19400-01	3.23000-01	-1	1.42300-01	-2.05250-03	-2.05250-03	03	-5.56425	-03	-1.55150-01	CC
1.92370-04	-0.14280-03	-2.58240-03	6.71760-03	-03	-2.63210-03	-2.63210-03	-2.63210-03	04	-5.34105	-04	-3.02670-02	CC
4.13220-03	-1.74000-03	-2.04290-03	2.47600-01	-03	-1.08270-01	1.08270-01	1.08270-01	02	1.58155	-02	-1.53510-00	CC
1.37190-04	6.54150-03	-1.51520-00	1.71420-03	-03	1.71420-03	1.71420-03	1.71420-03	03	-6.66035	-03	-1.58440-01	CC
1.03260-04	-5.61310-03	-3.33300-01	-4.43590-01	-03	6.43040-02	6.43040-02	6.43040-02	04	-6.39975	-04	-2.57540-01	CC

THE MATRIX ELEMENT  $\langle \text{EXF}(-\epsilon, 346440) - \text{C3T45N} \rangle$  2.0096E-011

[illegible]







THE MATRIX COEFFICIENTS OF EXP(-5.187805D 00)T+COS(2.532566D 00)T

1.054400-03	2.41020-04	-2.03810 00	6.07840-03	6.71500-01	1.12170-01	1.74120-03	-1.22450-02
2.14900-04	-6.54800-05	-8.31810-02	4.68810-04	-6.65910-02	5.04200-03	-2.15630-02	-3.86270-03
-4.52260-04	-6.03550-05	5.54100-01	-3.73960-03	6.82410-04	-5.58870-02	-3.05210-04	5.38200-03
1.16140-04	2.42230-03	-2.30060-01	7.27280-04	5.92790-02	1.29560-02	1.73330-04	-1.23450-03
1.92390-04	1.74330-03	-2.40370-03	-3.53690-03	9.99740-01	-1.22730-02	2.51150-05	-4.65560-02
3.12380-05	4.15460-06	-8.41190-02	-3.28380-04	5.27850-03	5.16370-03	-3.84300-04	1.26440-02
-1.01800-03	1.25620-04	2.2890 00	-8.88910-03	7.07750-02	-1.37620-01	-3.84300-04	-5.83670-04
7.23250-05	-2.22460-05	-1.92530-01	5.10920-04	-7.99350-02	1.28300-02	-3.83380-05	

THE MATRIX COEFFICIENTS OF EXP(-5.187805D 00)T+SIN(2.532566D 00)T

1.24330-04	1.81230-04	-3.25460-01	-9.54280-04	9.27170-01	5.14590-03	1.43170-03	-2.54540-03
-1.40080-04	3.83610-05	-2.52940-01	6.40570-04	1.31670-01	1.78160-02	-2.84370-04	1.57630-03
-6.01880-04	1.56610-04	5.66940-01	-1.51020-03	-7.81480-01	-4.78180-02	-1.45220-03	-5.53340-04
5.93220-03	-3.60480-05	-7.55680-02	-1.31650-05	1.21700-01	2.89270-01	-2.03660-04	1.54730-02
-1.24590-03	2.28240-04	-2.54370-02	-7.21670-03	-9.96610-01	-1.37550-01	-2.35520-03	-1.54730-02
1.24220-05	1.83260-05	-2.75830-02	1.58210-04	7.12820-02	4.76170-03	-3.38250-04	-2.33340-04
-1.46570-03	-4.71130-03	2.44950 00	-4.88560-03	-1.85100-02	-1.18570-03	-3.56800-03	1.55780-02
2.24120-04	6.43330-05	-3.56330-01	9.45390-04	2.31010-01	2.02530-02	4.80540-04	-2.50240-03

THE COMPLEX VECTOR IS PHASE (DEG)

1.10310800-03	-1.7058200E 01
1.42317370-04	-7.9891970E 01
7.5286370E-04	1.2821030E 02
1.30411720-04	-2.7071690E 01
1.35554330-03	8.18687190E 01
6.38598860-05	-5.72031450E 01
1.788455290-03	1.2478230E 02
2.345501300-04	-7.2114735E 01

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THE GAIN MATRIX G

0.0	0.0	4.50000-01	8.50000-01	0.0	0.0	0.0	0.0
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THE MODIFIED PLANT MATRIX, 4-FC

-1.70900-03	-1.44200-03	8.13480 00	-2.11880 01	7.69300-01	-2.73500-03	2.23100-03	0.0
-5.07300-03	-2.36600-01	3.72700 00	-7.39250 00	6.72600-01	6.57500-01	-1.73800-02	0.0
4.29270-04	-3.19100-04	-6.85220 00	-7.95260 03	-1.98100 00	1.68000-01	-1.03400-03	0.0
0.0	0.0	1.00000 00	0.0	0.0	0.0	0.0	0.0
-4.39900-03	-1.70600-03	5.53520 00	-9.89400-01	-7.71200 00	-2.74800-01	-1.31500-02	0.0
3.52300-05	-1.93000-03	2.51650-01	2.26950-01	2.04500-01	-3.66100-01	-1.23700-02	0.0
-2.04000-03	-1.42400-03	-1.47800 00	-4.16250 00	-2.47100 00	5.60800-01	-2.60500-02	0.0
0.0	0.0	0.0	0.0	1.33000 03	0.0	0.0	0.0

\*\*\*\*\*

THE CHARACTERISTIC POLYNOMIAL = IN ASCENDING POWERS OF S

1.01890360-04	5.25866260-01	3.35335660 00	1.16173750 01	4.80065370 01	1.66346230 02
8.18228910E 01	1.5158450E 01	1.00000000E 00			

\*\*\*\*\*





THE EIGENVALUES OF THE A MATRIX  
REAL PART  
IMAGINARY PART

-1.0399244E-04  
-3.8195892E-02  
3.8195892E-02  
-2.4473216E-01  
0.0  
-9.7459679E-01  
-6.7874815E-00  
-6.7874815E-00

\*\*\*\*\*

THE ELEMENTS OF THE STATE TRANSITION MATRIX

THE MATRIX COEFFICIENT OF EXP(-1.9399244E-04)t

9.92160-01 1.3650E-02 -2.5814E-00 -2.0309E-01 -8.5330E-01 -1.2600E-00 -7.3176E-05 -1.2146E-01  
-2.4953E-02 -3.4345E-04 -6.4420E-02 -5.1080E-01 -2.1366E-02 -3.1690E-02 -1.8905E-05 -3.0545E-01  
-1.3013E-09 -1.8002E-11 -3.3619E-09 -2.6680E-08 -1.3055E-09 -1.6538E-05 -1.7052E-13 -1.5929E-08  
-6.7072E-06 -9.2316E-08 -1.7316E-05 -1.3730E-04 -5.7462E-06 -0.5179E-06 -4.5475E-14 -8.2113E-08  
-1.4606E-09 -2.0017E-11 -4.1820E-05 -3.0641E-08 -1.0978E-09 -1.8894E-06 -5.6843E-14 -1.7852E-08  
-6.4219E-03 -8.8350E-05 -1.6370E-02 -1.3146E-01 -5.5037E-03 -8.1556E-02 -4.7362E-07 -7.8621E-02  
-1.9337E-01 -2.6615E-03 -4.5970E-01 -3.5583E-00 -1.6572E-01 -2.4557E-01 -1.4262E-05 -2.3673E-00  
7.5290E-06 1.6331E-07 -1.9438E-05 -1.5412E-04 -6.4525E-06 -9.5616E-06 -6.6569E-10 -5.2177E-05

THE MATRIX COEFFICIENT OF EXP(3.8195892E-02)t\*CS( 2.767631E-01)t

9.7865E-03 -6.2353E-03 -3.5117E-02 -5.6468E-00 -9.9410E-01 -9.0151E-02 -5.4526E-02 -8.1437E-00  
-3.8635E-03 -2.0767E-03 -1.5447E-02 -1.7697E-00 -3.1902E-01 -9.2747E-02 -2.0354E-02 -2.6428E-00  
3.6521E-05 -1.5759E-05 -2.2340E-04 -1.2155E-02 -2.2874E-03 -1.7328E-04 -1.8464E-04 -1.5307E-02  
-1.6781E-04 -2.6113E-05 -5.2371E-03 -6.0582E-02 -8.2305E-03 -7.4850E-03 -5.5561E-04 -5.7357E-02  
-4.5975E-04 -4.7873E-07 -5.2981E-02 -8.5440E-02 -9.8720E-03 -1.7125E-02 -1.5915E-03 -5.6140E-02  
4.5915E-03 -4.7469E-04 -5.3546E-02 -1.3782E-00 -1.8006E-01 -1.6938E-01 -8.7115E-02 -7.2139E-00  
2.0102E-01 -2.0357E-03 -1.5182E-00 -1.1619E-01 -5.8352E-02 -5.4865E-01 -1.7115E-01 -7.7139E-00  
6.8289E-05 -5.9700E-04 -5.4520E-03 -6.5412E-01 -1.1211E-01 -3.6318E-02 -1.7582E-01 -8.7634E-01

THE MATRIX COEFFICIENT OF EXP(3.8195892E-02)t\*SI( 2.767631E-01)t

-1.7395E-02 -2.2111E-03 -2.0711E-01 -5.7092E-00 -7.6062E-01 -7.4551E-01 -6.3804E-02 -5.2199E-00  
5.6844E-03 -9.3559E-04 -6.5930E-02 -2.1229E-00 -2.8988E-01 -2.5843E-01 -2.0354E-02 -2.0277E-00  
-4.2036E-05 -9.5509E-06 -5.4371E-04 -1.8769E-02 -3.6371E-03 -2.0871E-03 -1.5363E-04 -1.8841E-02  
-1.0894E-05 -6.0650E-05 -5.2640E-04 -5.2424E-02 -9.4008E-03 -4.0250E-04 -5.8513E-05 -7.7677E-02  
-8.8230E-05 -1.6831E-04 -8.0647E-04 -1.8342E-01 -3.0257E-02 -8.4437E-03 -7.7052E-04 -2.3535E-01  
2.2499E-03 -1.6231E-03 -6.5880E-03 -1.5260E-00 -2.6536E-02 -3.3920E-02 -1.3018E-02 -2.1551E-00  
-7.8159E-02 -6.4127E-02 -5.5478E-01 -8.0757E-01 -1.2725E-01 -5.3080E-00 -4.5983E-02 -5.8031E-01  
-1.8151E-03 -8.0062E-05 -1.8301E-02 -2.2737E-01 -2.0157E-02 -5.6589E-02 -7.4331E-03 -8.3616E-02

THE COMPLEX EIGENVECTOR IS  
MAGNITUDE PHASE (DEG)

1.9558628E-02 6.063700E-01  
6.8730464E-03 -1.242021E-02  
5.5662504E-05 4.8955513E-01  
1.9523126E-04 -3.146402E-01  
5.0747518E-04 1.6688774E-02  
5.1134442E-03 -2.6103421E-01  
2.0289519E-03 7.7590062E-00  
1.8163517E-03 8.7645422E-01





THE MATRIX COEFFICIENT OF EXP(-2.4473220-CU)

-1.05250-03	-2.97370-02	-1.90370-02	2.1639E-01	-3.13150-02	-1.04850-01	1.65800-03	-2.23410-01
3.78950-02	1.03120-00	6.225480-01	-7.52120-00	1.08740-00	3.63600-00	-5.88620-02	7.74730-00
-2.48910-06	6.77250-05	4.10700-05	-4.53960-04	-7.14180-05	2.53800-04	-3.86710-02	5.08810-04
-1.01670-05	-2.76130-04	-1.07850-04	2.01840-05	-2.91820-04	-9.35700-03	-1.58000-03	-2.07500-03
-1.99780-07	5.43300-06	3.29740-06	-2.56610-05	7.73470-06	-1.51740-05	-3.10510-07	4.08550-05
-2.89240-07	-7.70140-06	-2.97130-06	5.61710-05	-8.12130-03	-2.71550-02	-4.35150-04	-5.78590-02
3.12810-03	8.51460-02	5.16450-02	6.21020-01	8.57880-02	3.60220-01	-4.86150-03	6.39590-01
-9.16330-07	-2.22500-05	-1.34780-05	1.62060-04	-2.34320-05	-7.83470-05	1.26880-02	-1.66540-04

THE MATRIX COEFFICIENT OF EXP(-4.7775410-CU)

9.37490-03	2.85670-02	-6.14300-01	-1.05450-01	9.24750-01	3.40080-00	-6.79860-02	4.58220-00
-1.11970-02	-3.41420-02	7.33670-01	1.25940-01	-1.10440-00	-4.66160-00	5.11580-02	-5.47260-00
-6.55520-05	-2.12900-04	4.55700-03	-7.82320-02	-6.84300-03	-2.53100-02	5.64320-03	-3.39550-02
-1.45580-04	4.43200-04	-5.53950-03	1.63750-01	1.43600-02	5.23110-02	-1.06560-03	7.11570-02
-1.32420-04	-4.03700-04	6.77700-03	1.48950-01	-1.30220-02	-4.60360-02	-5.60310-04	-6.47240-02
-2.47150-03	7.53340-03	1.61950-01	-2.78000-00	2.43600-01	8.65560-01	-1.79240-02	1.20800-00
-2.27500-02	-6.93110-02	1.49070-00	2.55890-01	-2.24410-00	-8.25260-00	1.64580-01	-1.11200-01
2.77170-04	8.45180-04	-1.81620-02	-3.11760-01	2.73340-02	1.00550-01	-2.01010-02	1.35470-01

THE MATRIX COEFFICIENT OF EXP(-5.7453680-CU)

-1.07890-02	-6.40370-03	4.72770-00	3.73710-01	-1.50840-00	-2.00040-00	1.05550-02	-3.50050-01
-1.97820-03	1.17470-03	-8.67840-01	-6.85220-00	2.76570-01	3.66780-01	-1.42710-03	6.41840-02
-3.38180-04	-2.03620-04	1.48190-01	-1.17140-00	4.72820-02	6.27040-02	-3.32110-04	1.05730-02
-3.47000-04	-2.06000-04	1.52060-01	1.20200-00	-4.85150-02	-6.43380-02	-3.40770-04	-1.12590-02
-3.66660-04	2.05660-04	-1.51910-01	-1.20080-00	4.84670-02	6.42750-02	-3.40430-04	-1.12480-02
-3.72620-04	-2.21270-04	1.63280-01	1.29070-00	-5.20970-02	-6.50890-02	-3.65530-04	-1.20500-01
-1.22110-02	7.25130-03	-5.35090-00	-4.22980-01	1.70730-00	2.26410-00	-1.19520-02	3.96200-01
-3.55650-04	-2.11220-04	1.55870-01	1.23210-00	-4.97310-02	-6.55550-02	-3.45310-04	-1.15410-02

THE MATRIX COEFFICIENT OF EXP(-6.7874820-CU)

5.62930-04	1.41610-04	-1.57300-00	-1.39300-00	4.71230-01	5.46380-03	1.35620-03	-5.55500-03
-1.50650-04	-4.20200-05	-5.32290-01	-5.01000-01	3.80760-02	1.70110-02	-1.52510-04	-1.57410-03
-3.07640-04	1.02460-05	1.17430-00	1.10590-00	-4.26710-02	-3.78850-02	-3.32200-04	3.20660-03
5.14780-05	1.24200-05	-1.54430-01	-1.05550-01	4.26720-02	5.60930-02	-5.65850-05	-5.44540-04
2.85310-04	1.52570-04	1.48520-01	1.14130-00	5.54720-01	8.67490-04	1.37050-03	-3.17530-03
-1.40730-05	5.20030-05	-6.47870-02	-7.65630-02	-5.06420-03	-2.17360-03	3.64630-04	-2.44350-03
-2.41650-04	3.14830-05	1.33120-00	1.75250-00	3.39630-01	-1.33160-05	-1.67890-04	-2.64350-03
-3.51850-06	-1.47370-05	-1.22220-01	-2.22210-01	-8.56010-07	3.31050-05	-5.07370-04	-1.64270-05

THE MATRIX COEFFICIENT OF EXP(-6.7974820-CU)

1.22550-04	1.30570-04	5.65950-01	1.28240-00	7.44740-01	-1.21110-02	3.70070-04	-1.44070-03
-1.13630-04	6.08480-05	-6.51400-02	-2.31730-01	2.86140-01	3.68890-03	-4.38570-04	-1.24510-03
-2.64130-04	-1.33340-04	-2.12700-01	-4.23800-01	-6.18600-01	-1.07400-05	-5.66600-04	-2.80590-03
1.15920-05	-1.23860-05	4.56100-02	1.67700-01	6.85190-02	-1.00810-05	-8.95230-05	-1.36580-04
-5.95190-04	1.20330-04	1.284700-00	1.55740-00	-3.23610-01	-5.40840-02	-6.55000-04	-6.25000-03
2.21480-05	9.04440-06	-2.47010-02	5.72940-03	3.51150-02	1.27780-03	-6.65050-05	-2.38540-04
-5.50180-05	-2.04620-04	1.01730-00	1.36100-01	-8.54180-01	-3.82200-02	-1.51810-03	-5.90530-03
8.58220-05	-3.56120-05	-2.67870-01	-1.05380-01	5.60210-02	7.41570-03	-1.54360-04	-6.13340-03

THE COMPLEX EIGENVECTOR IS  
THE MAGNITUDE

5.76114560-04	-1.22812110-01
1.88598930-04	-3.70246720-01
4.05478790-04	-1.35300570-02
5.27662440-05	-1.26504950-01
6.60303100-04	-6.92887400-01
2.62408640-05	-5.75676600-01
5.00501730-04	-1.12712370-02
8.58937630-05	-9.76523120-01



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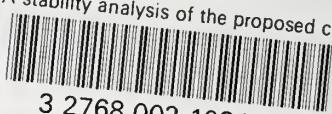
C514 Cline

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